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Abstract

In this paper, we consider resale in efficient auctions. The potential gains from trade arise from a delay in resale which reduces the bidders' values. We consider two information states during resale: (a) complete information where all the bids are revealed and (b) incomplete information where no bids are revealed. Under complete information, we establish revenue equivalence between the first- and second-price auction for a family of trade rules where the market power is distributed between the reseller and buyer. We also show that, if all the market power lies with the reseller (resp., buyer), it is optimal (resp., not) to reveal information.

JEL classification: D44, D82

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1 Introduction

Consider an efficient auction for one indivisible object with two bidders. Can there be potential gains from resale where the object is reallocated from the winner to loser? The answer is yes if the winner's value exceeds the loser's value in the primary market while the loser's value exceeds the winner's value in the secondary market. In other words, the winner's value declines faster than the loser's value when one moves from primary

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market to secondary market.[1] This phenomenon occurs when there is a delay between auction and resale during which the winner's value declines as he consumes the object while the loser's value declines as the object depletes.

The driving factors that induce resale after a delay where the values may decline disproportionately are limited purpose, dynamic market conditions, first user advantage, etc. These objects range from cars, books, furniture, equipment, technology, and music albums to art works, historic artifacts, writing instruments, limited editions, vintage items, and timepieces, to name a few.

For example, after buying a book, a buyer can generate almost all the value in a short period of time while depleting it very little. A buyer can resell an older technology to someone who does not have access to a superior technology. A buyer buys a limited edition car because he wants to use it before anyone else, and later he resells it. In all these examples, the winner's value declines faster than the loser's value.

Private-value auctions with resale has its origin based on the doctrine that the first-price auction is inefficient. In this paper, we consider resale in efficient auctions. To the best of our knowledge, this is the first paper to consider secondary markets in efficient auctions with private-values.

Consider two risk neutral bidders who draw their values for one indivisible object from a symmetric probability distribution. The design of a two-date auction game with a delayed resale is as follows. At date 1, which is called the bid date, either a first- or a second-price auction occurs. After date 1 and before date 2, which is called the interim date, the auction's winner consumes and depletes the object while the seller decides whether to reveal all the bids. As a result, bidders' value declines during the interim time. At date 2, which is called the resale date, bidders may trade the object via a trade rule.

Information concerning the revelation of bids after the bid date plays a vital role. If all the bids are revealed, then the bidders play a game under complete information during the resale date where valuations are common knowledge.[2] If no bids are revealed, then we are dealing with a game under incomplete information where the bidders trade based on the information regarding the ordinal rank of values.

Under complete information, we consider a family of trade rules where the market power is distributed between the two bidders. The two extreme rules are of particular importance. At one extreme, all the market power lies with the winner who extracts all the surplus from the loser. This rule is called the monopoly rule. At the other extreme, all the mar-

[1]This is the primary reason for the existence of secondary markets for second-hand objects.

[2]This follows from considering monotone bid functions.

ket power lies with the loser who extracts all the surplus from the winner. This rule is called the monopsony rule.

In this paper, we are interested in the following questions:

1. Under complete information, what is the impact of secondary markets on the bidders and the seller?
2. Will the seller benefit from information revelation?

The impact of the existence of secondary markets is two-fold: option value effect and partial cost recovery effect. If the bidder contemplates about the possibility to buy the object in the secondary market, his willingness to pay reduces which lowers the price of the object. This impact is captured by the option value effect. If the bidder contemplates about reselling the object, his willingness to pay rises which raises the price of the object. This impact is captured by the partial cost recovery effect. The net effect on the object's price will be determined by which effect is dominant.

Consequently, the bidders will make adjustments in their bids contingent on whether they act as resellers or buyers in the secondary market. The bid adjustment is the difference between economic rents incurred as a reseller and a buyer. If the bid adjustment is positive, the partial cost recovery effect dominates which raises the price. If the bid adjustment is negative, the option value effect dominates which reduces the price.

The sign of the bid adjustment will depend on the bidders' market power. Specifically, the bid adjustment is increasing in market power. It turns out that, in the case of monopoly, the bid adjustment is positive while in the case of monopsony, the bid adjustment is negative. So, bidders bid higher under the monopoly rule and lower under the monopsony rule due to the existence of secondary markets. As a result, the seller benefits from the presence of secondary markets in case of monopoly while the seller loses in case of monopsony. A more general result has been captured in Propositions 3, 6 and Corollaries 2, 3.

It is well-known that, regardless of the nature of probability distributions, there exists a dominant strategy where bidders bid their own values under second-price auction without resale. This property is not robust in the presence of secondary markets. However, the aforesaid strategy constitutes an equilibrium under resale without delay. In contrast, the present paper shows that bidders outbid their values under the monopoly rule and shade their values under the monopsony rule (Remark 4).

One of the most celebrated property in symmetric auctions – the revenue equivalence principle – says that the seller's expected revenues are equivalent under the first- and second-price auction. However, the principle breaks under asymmetry. In the absence of resale, there is no general revenue ranking principle while in the presence of resale without delays, there is a general revenue ranking principle which establishes that the first-price auction is revenue superior to the second-price auction.

We reestablish the revenue ranking principle under a delayed resale (Theorem 1). The intuition is based on the bid adjustment process discussed above. Importantly, the bid adjustment does not depend on the auction format due to complete information during resale. Consequently, the adjustment in the object's price equals the bid adjustment which leads to revenue equivalence principle.

Finally, we capture the impact of information. Consider the monopoly rule and the first-price auction. From the bid adjustment process, it can be argued that economic rents as a reseller are largest and as a buyer are smallest. Hence, bid adjustment is largest. The economic rents of reseller decline while that of buyer rise under incomplete information. Hence, the bid adjustment is lower for incomplete information case than for complete information case. Therefore, bidders' willingness to pay is higher under complete information. So, the seller prefers to reveal information. Using similar reasoning, we can argue that the seller prefers not to reveal information under the monopsony rule. These properties are captured in Corollaries 5, 6, and 7.

There is a technical contribution to the literature as well. Krishna [10], in his book, shows that there does not exist an equilibrium of the first-price auction in monotone strategies under complete information. In contrast, we show that under a delayed resale with complete information, there exists a unique equilibrium of the first-price auction in monotone strategies (Corollary 1).

1.1 The literature

The literature has relied on an implicit assumption that there is no delay between auction and resale. Khurana [8] is the first paper to consider fixed time delays in resale under asymmetric auctions.

Haile [6] and Garratt and Tröger [2] consider resale possibilities under symmetric auctions. Haile [6] considers symmetric auctions where the value of the object is not known to a bidder while submitting bids. Rather, bidders receive noisy signals about their values at the time of auction. During resale, they get additional information about their values which leads to expected potential gains from trade. Garratt and Tröger [2] also consider symmetric auctions but in their model one of the bidders is a speculator who has no value for the object. His sole purpose is to earn by reselling the object.

Gupta and Lebrun [3]; Hafalir and Krishna [4, 5]; Virág [15, 16]; Lebrun [11]; Cheng [1]; Zheng [17]; and Khurana [7, 9, 8] among others have considered resale possibilities under asymmetric auctions. Gupta and Lebrun [3] considers asymmetric bidders with complete information in the resale date. They derive a formula for the bid functions of the first-price auction.

Hafalir and Krishna [4, 5]; Virág [15, 16]; Lebrun [11]; Cheng [1]; and Khurana [7] consider asymmetric auctions with incomplete information during the resale date. With two risk neutral bidders, Hafalir and Krishna [4] show that bid symmetrization holds, i.e., the two bidders win with equal probability, and the first-price auction is revenue-dominant to the second-price auction. Virág [16] shows that bid symmetrization fails with two bidders if there are reserve prices. Virág [15] extends the analysis to more than two bidders and shows that bid symmetrization fails. Khurana [7] considers one risk neutral and one risk averse bidder and shows that bid symmetrization may or may not hold.

The paper is organized as follows. In Section 2, we setup the model. In Section 3, we characterize the first- and second-price auction under complete information and derive other properties. In Section 4, we characterize under incomplete information. In Section 5, we analyze the impact of information on the bidders and the seller. In Section 6, we conclude. The proofs are relegated to the appendix.

2 Economic model

Consider one unit of an indivisible object that has to be allocated via a first- or second-price auction. The set of two risk neutral bidders is denoted by $N = \{1, 2\}$. The values are drawn from a symmetric probability distribution $F : T \to \Re_+$, where $T = [0, \bar{a}] \subset \Re_+$ is the value space for both the bidders. We denote the random variables of the valuations for bidders 1 and 2 by $\mathcal{T}_1$ and $\mathcal{T}_2$ respectively. The probability distribution is twice continuously differentiable and the density function, denoted by f, is bounded away from zero. The seller is risk neutral and reserve prices are 0.

The structure of the game is as follows. Bidders play a two-date game, whereat date 1 – *the bid date*, the seller allocates the object via a first- or second-price auction. After date 1, there is a fixed time delay in the game that is exogenous. Thereafter, the game proceeds to date 2 – *the resale date*, where the two bidders engage in a resale trade. The game ends after date 2 and utilities are realized.

Next, we discuss about:

1. What happens during the interim time, i.e., between the two dates?
2. How much information is revealed after date 1?
3. What is the trade rule that is implemented at date 2?

During the interim time, the winner of date 1 consumes the object and obtains value from it while the loser loses value as the object depletes. The winner obtains and the loser loses value linearly with their own values. The parameter for winner is denoted by α_R and the parameter for loser is denoted by α_B, where $\alpha_R, \alpha_B \in (0, 1)$. For example, a bidder with value t obtains a value of $\alpha_R t$ in the interim time if he wins, and loses a value of $\alpha_B t$ if he loses. For the ease of exposition, we refer α_R as the *consumption rate* and α_B as the *depletion rate*.

The utilities under different circumstances in the first-price auction are as follows.

1. If a bidder with value t wins by bidding b and resells at p during the resale date, then his utility is $p + \alpha_R t - b$, where $\alpha_R t$ is the utility obtained by consuming the object in the interim.
2. If a bidder with value t wins by bidding b and does not resell during the resale date, then his utility is $t - b$.
3. If a bidder with value t loses and buys at p during the resale date, then his utility is $(1 - \alpha_B)t - p$, where $\alpha_B t$ is the utility lost in the interim.
4. If a bidder with value t loses and does not buy at the resale date, then his utility is 0.

On similar lines, we can define the utilities under second-price auction. An assumption that we follow throughout the paper is:

Assumption 1. $\alpha_R > \alpha_B$ *and* $\alpha_R + \alpha_B > 1$.

It says that the winner's value declines faster than the loser's value and these rates are sufficiently high.

We consider two informational cases:

1. Complete information: In this case, the seller reveals all the bids after date 1.
2. Incomplete information: In this case, the seller does not reveal any bid after date 1.

The complete information case has been discussed in Section 3 while the incomplete information case has been discussed in Section 4. As we will restrict to strictly increasing and continuous bid functions, the revelation of bids under the complete information case will lead to revelation of values. Therefore, the game turns into a game under complete information after date 1. While, under the incomplete information case, bidders update their beliefs about each other, conditional on winning and losing.

The trades rules for the two informational cases are discussed in Sections 3 and 4.

3 Complete information

In this section, we consider the complete information case where all the bids are revealed after date 1. Recall that, as bids functions will be strictly increasing and continuous, revealing bids imply that values are common knowledge at date 2. We consider a family of linear trade rules that is as follows. linear combination of winning and losing valuations. Formally, let the trade rule be

$$p(w,l) = \lambda_1 w + \lambda_2 l \tag{1}$$

where w is the value of the winner, l is the value of the loser and λ_1 and λ_2 are positive parameters. If a bidder with value w wins while a bidder with value l loses, then $p(w,l)$ is the payment that goes from the loser to the winner.

For notational convenience, let

$$k_1 = \frac{1-\alpha_R}{\lambda_2}, k_2 = \frac{\lambda_1}{1-\alpha_B}, k_3 = \max\left\{\frac{1-\alpha_R-\lambda_1}{\lambda_2}, \frac{\lambda_1}{1-\alpha_B-\lambda_2}\right\}$$

To ensure that there are potential gains from trade, we assume the following.

Assumption 2. *Either of the following must be true:*

1. $\lambda_1 = 0$ *and* $1-\alpha_R < \lambda_2 \leq 1-\alpha_B$.

2. $\lambda_2 = 0$ *and* $1-\alpha_R \leq \lambda_1 < 1-\alpha_B$.

3. $\lambda_1, \lambda_2 > 0$, $1-\alpha_R > \lambda_1$, $1-\alpha_B > \lambda_2$ *and* $k_3 < 1$.

At one extreme where $\lambda_1 = 0$ and $\lambda_2 = 1-\alpha_B$, the winner extracts all the surplus from the loser. Thus, we refer to this rule as a *monopoly rule*. At the other extreme where $\lambda_2 = 0$ and $\lambda_1 = 1-\alpha_R$, the loser extracts all the surplus from the winner. Thus, we refer to this rule as a *monopsony rule*.

In Subsection 3.1, we characterize all the equilibria of the first-price auction. In Subsection 3.2, we characterize all the equilibria of the second-price auction. In Section 3.3, we establish the revenue equivalence theorem.

3.1 First-price auction

Consider the first-price auction. Denote the symmetric bid function, that belongs to the family of strictly increasing, continuous and onto functions by β^1, i.e., $\beta^1 : T \to [0, \bar{b}^1]$ where $\bar{b}^1 > 0$ is the maximum bid. Let the symmetric inverse bid function be π^1, i.e., $\pi^1 : [0, \bar{b}^1] \to T$. Note that $\beta^1(0) = \pi^1(0) = 0$ and $\beta^1(\bar{a}) = \bar{b}^1$ and $\pi^1(\bar{b}^1) = \bar{a}$.

In the following result, we derive utility functions of bidders. Denote the expected utility function of a bidder in the first-price auction by $U^1 : T \times [0, \bar{b}^1] \to \Re$.

Proposition 1. *Consider a first-price auction under complete information. Let Assumptions 1 and 2 hold. The expected utility functions of a bidder with value t and bid b under different situations are as follows.*

If $\lambda_1 = 0$ and $1 - \alpha_R < \lambda_2 \leq 1 - \alpha_B$, then

$$\begin{aligned} U^1(t,b) = F(k_1 t)(t-b) + \int_{k_1 t}^{\pi^1(b)} (\alpha_R t + \lambda_2 \omega - b) f(\omega) \mathrm{d}\omega \\ + (1 - \alpha_B - \lambda_2) t [F(t/k_1) - F \circ \pi^1(b)] \end{aligned} \tag{2}$$

If $\lambda_2 = 0$ and $1 - \alpha_R \leq \lambda_1 < 1 - \alpha_B$, then

$$\begin{aligned} U^1(t,b) = (1 - \alpha_R - \lambda_1) t F(k_2 t) + F \circ \pi^1(b)[(\alpha_R + \lambda_1)t - b] \\ + \int_{\pi^1(b)}^{t/k_2} [(1-\alpha_B)t - \lambda_1 \omega] f(\omega) \mathrm{d}\omega \end{aligned} \tag{3}$$

If $\lambda_1, \lambda_2 > 0$, $1 - \alpha_R > \lambda_1$, $1 - \alpha_B > \lambda_2$ and $k_3 < 1$, then

$$\begin{aligned} U^1(t,b) = F(k_3 t)(t-b) + \int_{k_3 t}^{\pi^1(b)} [(\alpha_R + \lambda_1)t + \lambda_2 \omega - b] f(\omega) \mathrm{d}\omega \\ + \int_{\pi^1(b)}^{t/k_3} [(1 - \alpha_B - \lambda_2)t - \lambda_1 \omega] f(\omega) \mathrm{d}\omega \end{aligned} \tag{4}$$

For notational convenience, let $p^1(\pi^1(b), \pi^1(b)) = p^1(b)$, $\alpha = (\alpha_R, \alpha_B)$, $\lambda = (\lambda_1, \lambda_2)$ and $E(\alpha, \lambda) = 2\lambda_1 + 2\lambda_2 + \alpha_R + \alpha_B - 1$. The following result characterizes all the perfect Bayesian equilibria of the first-price auction under complete information.

Proposition 2. *Let Assumptions 1 and 2 be satisfied. A pair (π^1, p^1) is a symmetric perfect Bayesian equilibrium in monotone strategies under complete information if and only if it solves:*

$$\begin{aligned} \frac{F \circ \pi^1(b)}{\mathrm{D}F \circ \pi^1(b)} &= (\alpha_R + \alpha_B - 1)\pi^1(b) + 2p^1(b) - b \\ p^1(b) &= (\lambda_1 + \lambda_2)\pi^1(b) \end{aligned} \tag{5}$$

The following result provides a formula of the bid function for general probability distributions which ensures the existence of a unique equilibrium in symmetric and monotone strategies.

Corollary 1. *Let the primitives of Proposition 2 be true. Then, the bid function is characterized as*

$$\beta^1(t) = \frac{E(\alpha, \lambda)}{F(t)} \int_0^t \omega f(\omega) \mathrm{d}\omega \tag{6}$$

The function $E(\alpha, \lambda)$ captures the market power. A higher (resp., lower) value of E implies that the winner has more (resp., less) market power during resale.

Remark 1. *From Assumption 1,* $1+\alpha_B-\alpha_R \le E(\alpha,\lambda) \le 1+\alpha_R-\alpha_B$*. If the trade rule is monopoly, then* $E(\alpha,\lambda) = 1+\alpha_R-\alpha_B$*. If the trade rule is monopsony, then* $E(\alpha,\lambda) = 1+\alpha_B-\alpha_R$*. Clearly, bidders bid higher under the monopoly rule than the monopsony rule.*

Remark 2. *Given the monopoly rule, if the rate of consumption is high or the rate of depletion is low, bidders raise their bid. Given the monopsony rule, if the rate of consumption is low or the rate of depletion is high, bidders raise their bid.*

The following result compares β^1 with the standard symmetric independent private valuation model that is given in Riley and Samuelson [14] (henceforth, R-S). In other words, it captures the impact of resale with delays under complete information on the bid behavior. Let β^* be the bid function in the R-S model.

Proposition 3. *Let Assumptions 1 and 2 hold.*
1. *If* $E(\alpha,\lambda) > 1$*, then* $\beta^1(t) > \beta^*(t)$ *for every* $t \in (0,\bar{a}]$*.*
2. *If* $E(\alpha,\lambda) < 1$*, then* $\beta^1(t) < \beta^*(t)$ *for every* $t \in (0,\bar{a}]$*.*

The above result says that as long as $E(\alpha,\lambda) > 1$, bidders bid more aggressively in the presence of resale than they do when there are no resale possibilities. On the other hand, as long as $E(\alpha,\lambda) < 1$, bidders bid less aggressively in the presence of resale than they do when there are no resale possibilities.

Remark 3. *Under the monopoly rule, bidders bid higher than the case when resale is absent. Under the monopsony rule, bidders bid lower than the case when resale is absent. If either the monopoly or the monopsony rule is being implemented with* $\alpha_B \uparrow \alpha_R$*, then bids converge to the R-S model.*

To understand the intuition of the above result, we divide the impact of a delayed resale on bids into three effects: consumption effect, depletion effect, and market effect. The consumption effect captures the impact on bids due to the possibility of consuming the object before reselling it. The depletion effect captures the impact on bids due to depletion of the object. The market effect captures the impact on bids due to the market power that a bidder has during resale.

The consumption and depletion effects raise the bid as a bidder has an incentive to reduce his risk of losing. Thus, the consumption and depletion effects are positive. Under the monopoly rule, the market effect induces a bidder to raise his bid as the winner extracts all the loser's surplus. Thus, the market effect is positive. Under the monopsony rule, the market effect induces a bidder to lower his bid as the loser extracts all the winner's surplus. Thus, the market effect is negative.

Under the monopoly rule, the total effect induces a bidder to bid higher. Under the monopsony rule, the negative market effect dominates the positive consumption and depletion effects which reduces the bid.

Corollary 2. *Let the primitives of Proposition 3 hold. If $E(\alpha, \lambda) > 1$, the seller generates more expected revenues when resale happens after a delay under complete information than when there are no resale possibilities. If $E(\alpha, \lambda) < 1$, the seller generates less expected revenues when resale happens after a delay under complete information than when there are no resale possibilities.*

3.2 Second-price auction

In this subsection, we characterize all the equilibria of the second-price auction. Let β^2 be the symmetric bid function in the family of strictly increasing, continuous, and onto functions, i.e., $\beta^2 : T \to [0, \bar{b}^2]$ where $\bar{b}^2$ is the maximum bid. Let π^2 be the symmetric inverse bid function, i.e., $\pi^2 : [0, \bar{b}^2] \to T$.

In the following result, we derive utility functions of bidders. Denote the utility function of a bidder in the second-price auction by $U^2 : T \times [0, \bar{b}^2] \to \Re$.

Proposition 4. *Consider a second-price auction under complete information. Let Assumptions 1 and 2 hold. The expected utility functions of a bidder with value t and bid b under different situations are as follows.*

If $\lambda_1 = 0$ and $1 - \alpha_R < \lambda_2 \leq 1 - \alpha_B$, then

$$\begin{aligned} U^2(t,b) = \int_0^{k_1 t} [t - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{k_1 t}^{\pi^2(b)} [\alpha_R t + \lambda_2 \omega \\ - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + (1 - \alpha_B - \lambda_2) t [F(t/k_1) - F \circ \pi^2(b)] \end{aligned} \tag{7}$$

If $\lambda_2 = 0$ and $1 - \alpha_R \leq \lambda_1 < 1 - \alpha_B$, then

$$\begin{aligned} U^2(t,b) = \int_0^{k_2 t} [t - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{k_2 t}^{\pi^2(b)} [(\alpha_R + \lambda_1) t \\ - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{\pi^2(b)}^{t/k_2} [(1 - \alpha_B) t - \lambda_1 \omega] f(\omega) \mathrm{d}\omega \end{aligned} \tag{8}$$

If $\lambda_1, \lambda_2 > 0$, $1 - \alpha_R > \lambda_1$, $1 - \alpha_B > \lambda_2$ and $k_3 < 1$, then

$$\begin{aligned} U^2(t,b) = \int_0^{k_3 t} [t - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{k_3 t}^{\pi^2(b)} [(\alpha_R + \lambda_1) t + \lambda_2 \omega \\ - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{\pi^2(b)}^{t/k_3} [(1 - \alpha_B - \lambda_2) t - \lambda_1 \omega] f(\omega) \mathrm{d}\omega \end{aligned} \tag{9}$$

We skip the proof of the above proposition as it is based on similar line of that of Proposition 1.

For notational convenience, let $p^2(\pi^2(b), \pi^2(b)) = p^2(b)$. In the following result, we characterize the equilibria.

Proposition 5. *A pair (π^2, p^2) is a perfect Bayesian equilibrium of the second-price auction in symmetric and monotone strategies under complete information if and only if it solves:*

$$\pi^2(b) = \frac{b}{E(\alpha, \lambda)}, \quad p^2(b) = (\lambda_1 + \lambda_2)\pi^2(b) \tag{10}$$

Therefore, the bid function is

$$\beta^2(t) = E(\alpha, \lambda)t$$

Note that the above formula is *prior-free*, i.e., the formula is independent of the probability distribution. Furthermore, the above result ensure the existence of a unique equilibrium in symmetric and monotone strategies.

Remark 4. *If the trade rule is monopoly, the equilibrium is characterized as $\beta^2(t) = (1 + \alpha_R - \alpha_B)t$. As $\alpha_R > \alpha_B$, bidders bid more than their values, i.e., outbidding occur.*

If the trade rule is monopsony, the equilibrium is characterized as $\beta^2(t) = (1 + \alpha_B - \alpha_R)t$. As $\alpha_R > \alpha_B$, bidders bid less than their values, i.e., bid shading happens.

If $\alpha_B \uparrow \alpha_R$, the equilibrium converges to bid-your-own-value.

In the following result, we compare the bid functions of the second-price auction under complete information with the standard models.

Proposition 6. *Let Assumptions 1 and 2 hold.*

1. *If $E(\alpha, \lambda) > 1$, the bidders bid more aggressively during a delayed resale than under no resale.*
2. *If $E(\alpha, \lambda) < 1$, the bidders bid less aggressively during a delayed resale than under no resale.*

As it is well-known that bidders bid their value when resale is absent, the above result conveys that as long as $E(\alpha, \lambda) > 1$, bidders outbid their values and as long as $E(\alpha, \lambda) < 1$, bidders shade their values.

The intuition of the above result is similar to the intuition of Proposition 3.

Corollary 3. *Let the primitives of Proposition 2 hold. If $E(\alpha, \lambda) > 1$, the seller generates more expected revenue under a delayed resale with complete information than under no resale. If $E(\alpha, \lambda) < 1$, the seller generates less expected revenue under a delayed resale than under no resale.*

3.3 Revenue equivalence theorem

In this subsection, we compare a bidder's bid behavior and the seller's expected revenues between the first- and second-price auction.

The following result compares the bid function between the first- and second-price auction.

Proposition 7. *Let Assumptions 1 and 2 hold. Let (π^1, p^1) be a perfect Bayesian equilibrium of the first-price auction under complete information. Let (π^2, p^2) be a perfect Bayesian equilibrium of the second-price auction under complete information. Then,*

$$\pi^1(b) > \pi^2(b)$$

for every $b \in (0, \bar{b}^1]$.

The above result says that bidders bid more aggressively in the second-price auction than they do in the first-price auction.

It is well-known from Riley and Samuelson [14] and Myerson [13] that the seller's *ex-ante* expected revenues are equivalent in the first- and second-price auction as long as resale is absent and bidders are symmetric. In the case of asymmetric bidders and absence of resale, Maskin and Riley [12] show that a general revenue ranking principle does not exist for the two auction formats. Whenever the two bidders are asymmetric and resale occurs without a delay, Hafalir and Krishna [4] show that the first-price auction dominates the second-price auction in terms of expected revenues.

In contrast to the aforesaid results in the literature, the following result establishes a striking property that the expected revenues are equivalent in the two auction formats whenever bidders are symmetric and resale occurs after a fixed time delay.

Theorem 1. *Let Assumptions 1 and 2 be true. The seller's* ex-ante *expected revenues are equivalent in the first- and second-price auction under complete information and are given by:*

$$R = 2E(\alpha, \lambda) \int_0^{\bar{a}} t f(t)[1 - F(t)]\mathrm{d}t \tag{11}$$

The above result is called the *revenue equivalence theorem.*

Let us understand the intuition behind the revenue equivalence theorem. Due to the possibility of a delayed resale, two effects enforce in the opposite direction: *option value effect* and *partial cost recovery effect.* The option value effect captures the impact on price of the object due to an option of buying the object in the resale market. The partial cost recovery effect captures the impact on price of the object due to a possibility of recovering partial costs by reselling the object.

Clearly, the option value effect reduces the price of the object while the partial cost recovery effect raises the price. Whether the price rises or declines depends on which effect is dominant. If the partial cost recovery effect dominates the option value effect, then the price rises. Otherwise, it declines.

To capture which effect dominates, note that the net expected surplus from resale is the difference between expected surplus as a reseller and expected surplus as a buyer. If the net expected surplus is positive, then the partial cost recovery effect dominates the option value effect. If the net expected surplus is negative, the option value effect dominates.

Note that the net expected surplus rises with the reseller's market power. In case of a monopoly where the reseller has all the market power, the net expected surplus is positive, while in the case of a monopsony where the buyer has all the market power, the net expected surplus is negative.

As a result, a change in the price of the object due to the presence of resale equals the net expected surplus. Due to complete information during resale, the net expected surplus is independent of the selling mechanism and only depends on the market power. Thus, the net expected surplus' are equal in the first- and second-price auction which leads to the revenue equivalence theorem.

Corollary 4. *The seller's expected revenues are maximized under the monopoly rule.*

4 Incomplete information

Consider the incomplete information case where no bids are revealed. Unlike the complete information case, we cannot consider a family of trade rules incomplete information as values are private information during resale. To tract the essence of impact of market power, we consider two extreme cases: monopoly rule and monopsony rule.

To ensure the existence of a unique resale price, we assume that:

Assumption 3. *The hazard rate, $f/(1-F)$, is non-decreasing on the valuation space.*

Consider the first-price auction and monopoly rule. We restrict to the family of symmetric perfect Bayesian equilibria where the bid functions are strictly increasing, continuous, and onto. Let the inverse bid function and the resale price be σ^1 and q^1 respectively.

In the following result, we characterize all the perfect Bayesian equilibria of the first-price auction under the monopoly rule.

Proposition 8. *Let Assumptions 1 and 3 be satisfied. A profile (σ^1, q^1) is a perfect Bayesian equilibrium of the first-price auction under incomplete information and the monopoly rule if and only if it solves the following Dirichlet problem:*

$$
\begin{aligned}
\mathrm{D}\sigma^1(b) &= \frac{F \circ \sigma^1(b)}{f \circ \sigma^1(b)} \frac{1}{2q^1(b) + (\alpha_R + \alpha_B - 1)\sigma^1(b) - b} \\
(1-\alpha_R)\sigma^1(b) &= q^1(b) - \frac{F \circ \sigma^1(b) - F(zq^1(b))}{zf(zq^1(b))} \\
\sigma^1(0) = 0, &\quad \sigma^1(\hat{b}^1) = \bar{a} \quad \text{for some} \quad \hat{b}^1 > 0
\end{aligned}
\tag{12}
$$

where $z = 1/(1-\alpha_B)$.

Consider the first-price auction and the monopsony rule. Let the inverse bid function and the resale price be σ^1_* and q^1_* respectively. The next result characterizes all the perfect Bayesian equilibria of the first-price auction under the monopsony rule.

Proposition 9. *Let Assumptions 1 and 3 be satisfied. A profile (σ^1_*, q^1_*) is a perfect Bayesian equilibrium of the first-price auction under incomplete information and the monopsony rule if and only if it solves the following Dirichlet problem:*

$$
\begin{aligned}
\mathrm{D}\sigma^1_*(b) &= \frac{F \circ \sigma^1_*(b)}{f \circ \sigma^1_*(b)} \frac{1}{2q^1_*(b) + (\alpha_R + \alpha_B - 1)\sigma^1_*(b) - b} \\
(1-\alpha_B)\sigma^1_*(b) &= q^1_*(b) - \frac{F \circ \sigma^1_*(b) - F(yq^1_*(b))}{yf(yq^1_*(b))} \\
\sigma^1_*(0) = 0, &\quad \sigma^1_*(\hat{b}^1_*) = \bar{a} \quad \text{for some} \quad \hat{b}^1_* > 0
\end{aligned}
\tag{13}
$$

where $y = 1/(1-\alpha_R)$.

The proof of Proposition 9 is similar to the proof of Proposition 10. Therefore, we skip it.

Consider the second-price auction. Under the monopoly rule, if no bids are revealed, the winner will extract all the surplus from loser as he knows his value. Therefore, the equilibrium is equivalent to the one in Remark 5.

Now, consider the second-price auction and the monopsony rule. Let σ^2 be a symmetric inverse bid function, which belongs to the family of strictly increasing, continuous, and onto functions. Denote the resale price by q^2.

In the following result, we characterize the equilibria of the second-price auction under the monopsony rule.

Proposition 10. *Let Assumptions 1 and 3 be satisfied. A profile* (σ^2, q^2) *is a perfect Bayesian equilibrium in monotone strategies under incomplete information and the monopsony rule if and only if the following holds:*

$$\begin{aligned} \sigma^2(b) &= \frac{b - 2q^2(b)}{\alpha_R + \alpha_B - 1} \\ (1-\alpha_B)\sigma^2(b) &= q^2(b) - \frac{F \circ \sigma^2(b) - F(yq^2(b))}{yf(yq^2(b))} \end{aligned} \tag{14}$$

where $y = 1/(1-\alpha_R)$.

5 Impact of information

In this section, we capture the impact of information on the bidders and the seller. Specifically, we answer the question: when does the seller reveal information?

In the following two results, we compare the bid functions of the first-price auction between complete and incomplete information under the monopoly and monopsony rules.

Proposition 11. *Consider the monopoly rule and the first-price auction. Let* (π^1, p^1) *be a symmetric perfect Bayesian equilibrium in monotone strategies under complete information. Let* (σ^1, q^1) *be a symmetric perfect Bayesian equilibrium under incomplete information. Let Assumptions 1, 2, and 3 be satisfied and* $F(0) > 0$. *Then,*

$$\pi^1(b) < \sigma^1(b)$$

for every $b \in (0, \hat{b}^1]$.

The above result says that if the first-price auction and the monopoly rule are implemented, bidders bid higher under complete information than under incomplete information.

Corollary 5. *Consider the monopoly rule and the first-price auction. The seller's* ex-ante *expected revenues are higher under complete information than under incomplete information.*

Proposition 12. *Consider the monopsony rule and the first-price auction. Let* (π^1, p^1) *be a symmetric perfect Bayesian equilibrium in monotone strategies under complete information. Let* (σ^1_*, q^1_*) *be a symmetric perfect Bayesian equilibrium under incomplete information. Let Assumptions 1, 2, and 3 be satisfied and* $F(0) > 0$. *Then,*

$$\pi^1(b) > \sigma^1_*(b)$$

for every $b \in (0, \bar{b}^1]$.

The above result says that if the first-price auction and the monopsony rule are implemented, bidders bid lower under complete information than under incomplete information.

Corollary 6. *Consider the monopsony rule and the first-price auction. The seller's* ex-ante *expected revenues are lower under complete information than under incomplete information.*

The next result compares the bid functions of the second-price auction between complete and incomplete information under the monopsony rule.

Proposition 13. *Consider the monopsony rule and the second-price auction. Let (π^2, p^2) be a symmetric perfect Bayesian equilibrium in monotone strategies under complete information. Let (σ^2, q^2) be a symmetric perfect Bayesian equilibrium under incomplete information. Let Assumptions 1, 2, and 3 be satisfied. Then,*

$$\pi^2(b) > \sigma^2(b)$$

for every $b \in (0, \bar{b}^2]$.

The above result says that if the second-price auction and the monopsony rule are implemented, bidders bid lower under complete information than under incomplete information.

Corollary 7. *Consider the monopsony rule and the second-price auction. The seller's* ex-ante *expected revenues are lower under complete information than under incomplete information.*

Therefore, the seller reveals information under the monopoly rule and does not reveal information under the monopsony rule.

Let us understand the intuition behind the above properties. Recollect that the bid adjustment under complete information due to a delayed resale equals the economic rent as a reseller minus the economic rent as a buyer and it is independent of the auction format. As a reseller, the largest economic rents are extracted under the monopoly rule while the smallest economic rents are extracted under the monopsony rule. As a buyer, the largest economic rents are extracted under the monopsony rule while the smallest economic rents are extracted under the monopoly rule. So, the largest positive bid adjustment occurs under the monopoly rule while the largest negative bid adjustment occurs under the monopsony rule.

Under incomplete information and the monopoly rule, the reseller's economic rent declines and the buyer's economic rent rises which lead to a decline in bid adjustment. Hence, the seller benefits by revealing information.

Under incomplete information and the monopsony rule, the reseller's economic rent rises and the buyer's economic rent declines which lead to a rise in bid adjustment. Hence, the seller benefits by suppressing information.

6 Conclusion

In this paper, we have considered secondary markets in efficient auctions when an object is resold after a delay. Due to a delay, the winner's value declines as he consumes the object while the loser's value declines as the object depletes. The bidders' value declines disproportionately which leads to expected gains from resale.

We have considered two informational states: complete information where all bids are revealed post auction and incomplete information where no bids are revealed. Our main result established revenue equivalence between the first- and second-price auction under complete information for a family of trade rules that included the monopoly rule – where all the market power lies with the reseller and the monopsony rule – where all the market power lies with the buyer.

We have also shown the impact of information on the bidders and the seller. The main findings are that the seller benefits from revealing information under the monopoly rule and loses from revealing information under the monopsony rule.

A Appendix: Proofs

Proof of Proposition 1. We show 1. As the trade rule is exogenous, consider date 1 and bidder 1 with value t. He wins with bid b if and only if $\mathcal{T}_2 < \pi^1(b)$. Trade succeeds if and only if $(1-\alpha_R)t \leq \lambda_2\mathcal{T}_2$ and $(1-\alpha_B)\mathcal{T}_2 \geq \lambda_2\mathcal{T}_2$. The first condition says that the ex-post value of the winner is less than the resale price and the second condition says that the ex-post value of the loser must be more than the resale price. The two conditions together imply $\mathcal{T}_2 \geq k_1 t$. Otherwise, trade does not succeed. Therefore, with probability that $\mathcal{T}_2 < k_1 t$, trade does not happen and bidder 1 keeps the object which gives him a utility of $t - b$, and with probability that $k_1 t \leq \mathcal{T}_2 < \pi^1(b)$, trade happens which gives him a utility of $\alpha_R t + \lambda_2\mathcal{T}_2 - b$.

On the other hand, bidder 1 loses if and only if $\mathcal{T}_2 > \pi^1(b)$. In this case, trade succeeds if and only if $(1-\alpha_B)t \geq \lambda_2 t$ and $(1-\alpha_R)\mathcal{T}_2 \leq \lambda_2 t$. These imply $\mathcal{T}_2 \leq t/k_1$. Therefore, with probability that $\pi^1(b) < \mathcal{T}_2 \leq t/k_1$, trade happens thereby giving bidder 1 a utility of $(1-\alpha_B-\lambda_2)t$.

Thus, the expected utility function of bidder 1 is

$$
\begin{aligned}
U^1(t,b) = {} & \Pr(\mathcal{T}_2 < k_1 t)(t-b) + \Pr(k_1 t \le \mathcal{T}_2 < \pi^1(b))(\alpha_R t + \lambda_2 \mathcal{T}_2 - b) \\
& + \Pr(\pi^1(b) < \mathcal{T}_2 \le t/k_2)(1-\alpha_B-\lambda_2)t
\end{aligned}
$$

which can be rewritten as (2).

We show 2. As the trade rule is exogenous, consider date 1 and bidder 1 with value t. He wins with bid b if and only if $\mathcal{T}_2 < \pi^1(b)$. Trade succeeds if and only if $(1-\alpha_R)t \le \lambda_1 t$ and $(1-\alpha_B)\mathcal{T}_2 \ge \lambda_1 t$. These imply $\mathcal{T}_2 \ge k_2 t$. Otherwise, trade does not succeed. Therefore, with probability that $\mathcal{T}_2 < k_2 t$, trade does not happen and bidder 1 keeps the object which gives him a utility of $t-b$, and with probability that $k_2 t \le \mathcal{T}_2 < \pi^1(b)$, trade happens which gives him a utility of $(\alpha_R+\lambda_1)t-b$.

On the other hand, bidder 1 loses if and only if $\mathcal{T}_2 > \pi^1(b)$. In this case, trade succeeds if and only if $(1-\alpha_B)t \ge \lambda_1 \mathcal{T}_2$ and $(1-\alpha_R)\mathcal{T}_2 \le \lambda_1 \mathcal{T}_2$. These imply $\mathcal{T}_2 \le t/k_2$. Therefore, with probability that $\pi^1(b) < \mathcal{T}_2 \le t/k_2$, trade happens thereby giving bidder 1 a utility of $(1-\alpha_B)t - \lambda_1 \mathcal{T}_2$.

Thus, the expected utility function of bidder 1 is

$$
\begin{aligned}
U^1(t,b) = {} & \Pr(\mathcal{T}_2 < k_2 t)(t-b) + \Pr(k_2 t \le \mathcal{T}_2 < \pi^1(b))[(\alpha_R+\lambda_1)t - b] \\
& + \Pr(\pi^1(b) < \mathcal{T}_2 < t/k_2)[(1-\alpha_B)t - \lambda_1 \mathcal{T}_2]
\end{aligned}
$$

which can be rewritten as (3).

We show 3. As the trade rule is exogenous, consider date 1 and bidder 1 with value t. He wins with bid b if and only if $\mathcal{T}_2 < \pi^1(b)$. Trade succeeds if and only if $(1-\alpha_R)t \le p^1(t,\mathcal{T}_2) = \lambda_1 t + \lambda_2 \mathcal{T}_2$ and $(1-\alpha_B)\mathcal{T}_2 \ge p^1(t,\mathcal{T}_2)$. These imply $\mathcal{T}_2 \ge k_3 t$. Otherwise, trade does not succeed. Therefore, with probability that $\mathcal{T}_2 < k_3 t$, trade does not happen and bidder 1 keeps the object which incurs him a utility of $t-b$, and with probability that $k_3 t \le \mathcal{T}_2 < \pi^1(b)$, trade happens and bidder 1 gets a utility of $\alpha_R t + p^1 - b$.

On the other hand, bidder 1 loses if and only if $\mathcal{T}_2 > \pi^1(b)$. In this case, trade succeeds if and only if $(1-\alpha_B)t \ge p^1(\mathcal{T}_2,t) = \lambda_1 \mathcal{T}_2 + \lambda_2 t$ and $(1-\alpha_R)\mathcal{T}_2 \le p^1(\mathcal{T}_2,t)$. These imply $\mathcal{T}_2 \le t/k_3$. Therefore, with probability that $\pi^1(b) < \mathcal{T}_2 \le t/k_3$, trade happens thereby giving bidder 1 a utility of $(1-\alpha_B)t - p^1$.

Thus, the expected utility function of bidder 1 is

$$
\begin{aligned}
U^1(t,b) = {} & \Pr(\mathcal{T}_2 < k_3 t)(t-b) + \Pr(k_3 t \le \mathcal{T}_2 < \pi^1(b))[(\alpha_R+\lambda_1)t \\
& + \lambda_2 \mathcal{T}_2 - b] + \Pr(\pi^1(b) < \mathcal{T}_2 \le t/k_3)[(1-\alpha_B-\lambda_2)t - \lambda_1 \mathcal{T}_2]
\end{aligned}
$$

which can be rewritten as (4). ■

Proof of Proposition 2. We first show for the case when part 3 of Assumption 1 holds. In this case, $p^1(w,l) = \lambda_1 w + \lambda_2 l$. Suppose (π^1, p^1) is a symmetric perfect Bayesian equilibrium. We can write (4) as

$$\begin{aligned} U^1(t,b) = F(k_3 t)(t-b) + \int_{k_3 t}^{\pi^1(b)} [\alpha_R t + p^1(t,\omega) - b] f(\omega) \mathrm{d}\omega \\ + \int_{\pi^1(b)}^{t/k_3} [(1-\alpha_B)t - p^1(\omega,t)] f(\omega) \mathrm{d}\omega \end{aligned} \tag{15}$$

Applying Leibniz integral rule, the first-order derivative of (15) is

$$\begin{aligned} \mathrm{D}_b U^1(t,b) &= -F(k_3 t) + [\alpha_R t + p^1(t,\pi^1(b)) - b] \mathrm{D}F \circ \pi^1(b) - F \circ \pi^1(b) \\ &\quad + F(k_3 t) - [(1-\alpha_B)t - p^1(\pi^1(b),t)] \mathrm{D}F \circ \pi^1(b) \\ &= [(\alpha_R + \alpha_B - 1)t + p^1(t,\pi^1(b)) - b + p^1(\pi^1(b),t)] \\ &\quad \mathrm{D}F \circ \pi^1(b) - F \circ \pi^1(b) \end{aligned}$$

In equilibrium, $t = \pi^1(b)$ and $\mathrm{D}_b U^1(\pi^1(b), b) = 0$. This gives

$$\frac{F \circ \pi^1(b)}{\mathrm{D}F \circ \pi^1(b)} = (\alpha_R + \alpha_B - 1)\pi^1(b) + 2p^1(b) - b \tag{16}$$

Conversely, suppose (π^1, p^1) solves (5). We show that (π^1, p^1) is an equilibrium. Suppose bidder 1 with value t and bid b overbids to c where $\pi^1(c) > t$. Then, the derivative of $U^1(t,b)$ implies

$$\begin{aligned} \mathrm{D}_c U^1(t,c) &= [(\alpha_R + \alpha_B - 1)t + p^1(t,\pi^1(c)) - c + p^1(\pi^1(c),t)] \\ &\quad \mathrm{D}F \circ \pi^1(c) - F \circ \pi^1(c) \\ &= [(\lambda_1 + \lambda_2 + \alpha_R + \alpha_B - 1)t + (\lambda_1 + \lambda_2)\pi^1(c) - c] \\ &\quad \mathrm{D}F \circ \pi^1(c) - F \circ \pi^1(c) \\ &< [E(\alpha,\lambda)\pi^1(c) - c] \mathrm{D}F \circ \pi^1(c) - F \circ \pi^1(c) \\ &= 0 \end{aligned}$$

Therefore, overbids are not profitable. On similar lines, it can be shown that underbids are also not profitable.

We show for the case when part 1 of Assumption 1 holds. Suppose (π^1, p^1) is a symmetric perfect Bayesian equilibrium. Applying Leibniz integral rule, the first-order derivative of (2) is

$$\begin{aligned} \mathrm{D}U^1(t,b) = \mathrm{D}F \circ \pi^1(b)[\alpha_R t + \lambda_2 \pi^1(b) - b] - F \circ \pi^1(b) \\ - \mathrm{D}F \circ \pi^1(b)(1 - \alpha_B - \lambda_2)t \end{aligned}$$

Using $t = \pi^1(b)$ and $\mathrm{D}_b U^1(\pi^1(b), b) = 0$, we arrive at (16) with $\lambda_1 = 0$. On similar lines of part 3, we can show the converse.

We show for the case when part 2 of Assumption 1 holds. Suppose (π^1, p^1) is a symmetric perfect Bayesian equilibrium. Applying Leibniz integral rule, the first-order derivative of (3) is

$$\begin{aligned}\mathrm{D}U^1(t,b) &= \mathrm{D}F \circ \pi^1(b)[(\alpha_R + \lambda_1)t - b] - F \circ \pi^1(b) \\ &\quad - \mathrm{D}F \circ \pi^1(b)[(1-\alpha_B)t - \lambda_1 \pi^1(b)]\end{aligned}$$

Using $t = \pi^1(b)$ and $\mathrm{D}_b U^1(\pi^1(b), b) = 0$, we arrive at (16). On similar lines of part 3, we can show the converse. ■

Proof of Corollary 1. As $p^1(b) = (\lambda_1 + \lambda_2)\pi^1(b)$, from (5), we have

$$\mathrm{D}\pi^1(b) = \frac{F \circ \pi^1(b)}{f \circ \pi^1(b)} \frac{1}{E(\alpha, \lambda)\pi^1(b) - b}$$

As $b = \beta^1 \circ \pi^1(b)$ implies $1 = \mathrm{D}\beta^1 \circ \pi^1(b)\mathrm{D}\pi^1(b)$, we have

$$\frac{1}{\mathrm{D}\beta^1 \circ \pi^1(b)} = \frac{F \circ \pi^1(b)}{f \circ \pi^1(b)} \frac{1}{E(\alpha, \lambda)\pi^1(b) - b}$$

Using $t = \pi^1(b)$, we have

$$\frac{1}{\mathrm{D}\beta^1(t)} = \frac{F(t)}{f(t)} \frac{1}{E(\alpha, \lambda)t - \beta^1(t)}$$

This implies

$$E(\alpha, \lambda)tf(t) = \mathrm{D}[F(t)\beta^1(t)]$$

Using the fundamental theorem of calculus, we have

$$\beta^1(t) = \frac{E(\alpha, \lambda)}{F(t)} \int_0^t \omega f(\omega)\mathrm{d}\omega$$

■

Proof of Proposition 3. From Riley and Samuelson [14], the symmetric bid function is

$$\beta^*(t) = \frac{1}{F(t)} \int_0^t \omega f(\omega)\mathrm{d}\omega$$

Comparing this with (6), we have

$$\beta^1(t) > \beta^*(t)$$

for every $t \in (0, \bar{a}]$. ■

Proof of Proposition 5. We first show for the case when part 3 of Assumption 3 holds. In this case, $p^2(w,l) = \lambda_1 w + \lambda_2 l$. Suppose (π^2, p^2) is an equilibrium. We can rewrite (9) as

$$\begin{aligned} U^2(t,b) = \int_0^{k_3 t} [t - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{k_3 t}^{\pi^2(b)} [\alpha_R t + p^2(t,\omega) \\ - \beta^2(\omega)] f(\omega) \mathrm{d}\omega + \int_{\pi^2(b)}^{t/k_3} [(1-\alpha_B)t - p^2(\omega,t)] f(\omega) \mathrm{d}\omega \end{aligned} \tag{17}$$

Applying Leibniz integral rule while differentiating (17), we have

$$\mathrm{D}_b U^2(t,b) = \mathrm{D}F \circ \pi^2(b)[(\alpha_R + \alpha_B - 1)t + p^2(t, \pi^2(b)) - b + p^2(\pi^2(b), t)] \tag{18}$$

In equilibrium, $\pi^2(b) = t$ and $\mathrm{D}_b U^2(\pi^2(b), b) = 0$. As $\mathrm{D}\pi^2(b), f \circ \pi^2(b) > 0$, we have

$$\pi^2(b) = \frac{b - 2p^2(b)}{\alpha_R + \alpha_B - 1}$$

As $p^2(b) = (\lambda_1 + \lambda_2)\pi^2(b)$, we have

$$\pi^2(b) = \frac{b}{E(\alpha, \lambda)} \tag{19}$$

We show the converse. Suppose (π^2, p^2) solve (10). Consider bidder 1 with value t and bid b. Suppose he underbids to c such that $t > \pi^2(c)$. Then, $p^2(t, \pi^2(c)) > p^2(c)$ and $p^2(\pi^2(c), t) > p^2(c)$ and from (18), we have

$$\begin{aligned} \mathrm{D}_c U^2(t,c) &= \mathrm{D}F \circ \pi^2(c)[(\alpha_R + \alpha_B - 1)t + p^2(t, \pi^2(c)) - c + p^2(\pi^2(c), t)] \\ &> \mathrm{D}F \circ \pi^2(c)[(\alpha_R + \alpha_B - 1)\pi^2(c) + 2p^2(c) - c] \\ &= 0 \end{aligned}$$

Therefore, underbids are not profitable. Similarly, it can be shown that overbids are also not profitable.

We show for the case when part 1 of Assumption 1 holds. Suppose (π^2, p^2) is a symmetric perfect Bayesian equilibrium. Applying Leibniz integral rule, the first-order derivative of (7) is

$$\mathrm{D}_b U^2(t,b) = \mathrm{D}F \circ \pi^2(b)[(\alpha_R + \alpha_B + \lambda_2 - 1)t + \lambda_2 \pi^2(b) - b]$$

Using $t = \pi^2(b)$ and $\mathrm{D}_b U^2(\pi^2(b), b) = 0$, we arrive at (19) with $\lambda_1 = 0$. On similar lines of part 3, we can show the converse.

We show for the case when part 2 of Assumption 1 holds. Suppose (π^2, p^2) is a symmetric perfect Bayesian equilibrium. Applying Leibniz integral rule, the first-order derivative of (19) is

$$\mathrm{D}_b U^2(t,b) = \mathrm{D}F \circ \pi^2(b)[(\alpha_R + \alpha_B + \lambda_1 - 1)t + \lambda_1 \pi^2(b) - b]$$

Using $t = \pi^2(b)$ and $\mathrm{D}_b U^2(\pi^2(b), b) = 0$, we arrive at (19). On similar lines of part 3, we can show the converse. ■

Proof of Proposition 7. Pick an arbitrary $t > 0$. As $\int_0^t \omega f(\omega)\mathrm{d}\omega < \int_0^t t f(\omega)\mathrm{d}\omega = tF(t)$, we have

$$\frac{1}{F(t)} \int_0^t \omega f(\omega)\mathrm{d}\omega < t$$

which is equivalent to

$$\frac{E(\alpha,\lambda)}{F(t)} \int_0^t \omega f(\omega)\mathrm{d}\omega < E(\alpha,\lambda)t$$

Thus, $\beta^1(t) < \beta^2(t)$. ■

Proof of Theorem 1. Consider the first-price auction and bidder 1 with value t. The interim payments generated from him are

$$\begin{aligned} P^1(t) &= \beta^1(t)F(t) \\ &= E(\alpha,\lambda) \int_0^t \omega f(\omega)\mathrm{d}\omega \end{aligned}$$

The *ex-ante* expected revenues generated from bidder 1 are

$$\begin{aligned} \mathbb{E}[P^1] &= \int_0^{\bar{a}} P_1^1(t) f(t)\mathrm{d}t \\ &= E(\alpha,\lambda) \int_0^{\bar{a}} \int_0^t \omega f(\omega) f(t)\mathrm{d}\omega\mathrm{d}t \end{aligned}$$

where $\mathbb{E}$ is the expectation operator. Using Fubini's theorem, we have

$$\begin{aligned} \mathbb{E}[P^1] &= E(\alpha,\lambda) \int_0^{\bar{a}} \int_t^{\bar{a}} t f(t) f(\omega)\mathrm{d}\omega\mathrm{d}t \\ &= E(\alpha,\lambda) \int_0^{\bar{a}} t f(t)[1 - F(t)]\mathrm{d}t \end{aligned}$$

Therefore, the *ex-ante* expected revenues are

$$R^1 = 2E(\alpha,\lambda) \int_0^{\bar{a}} t f(t)[1 - F(t)]\mathrm{d}t$$

Now, consider the second-price auction and bidder 1 with value t. The interim payments generated from him are

$$\begin{aligned} P^2(t) &= \int_0^t \beta^2(\omega) f(\omega) \mathrm{d}\omega \\ &= E(\alpha, \lambda) \int_0^t \omega f(\omega) \mathrm{d}\omega \\ &= P^1(t) \end{aligned}$$

Therefore, the *ex-ante* expected revenues are

$$R^2 = 2E(\alpha, \lambda) \int_0^{\bar{a}} t f(t)[1 - F(t)] \mathrm{d}t$$

■

Proof of Proposition 8. We solve the game by backward induction. Consider the resale date and bidder 1 with value t. Since he chooses an optimum resale price, it must be true that he wins at date 1. Suppose he wins with a bid b. Then, it must be the case that $b > \mu^1(\mathcal{T}_2)$, which is equivalent to $\mathcal{T}_2 < \sigma^1(b)$.

In Lemma B.1 of Appendix B, we show that the winner always offers the object at date 2. Since bidder 1 wins, he offers the object to bidder 2 at price q^1. Bidder 2 accepts if and only if his utility at date 2 exceeds the resale price, i.e., $(1 - \alpha_B)\mathcal{T}_2 > q^1$ which is equivalent to $\mathcal{T}_2 > zq^1$, where $z = 1/(1 - \alpha_B)$. If $\mathcal{T}_2 < zq^1$, bidder 2 rejects the offer. Therefore, the expected utility function of bidder 1 is

$$\begin{aligned} U^1(t, b, q^1) = &\Pr[\mathcal{T}_2 > zq^1 | \mathcal{T}_2 < \sigma^1(b)](q^1 + \alpha_R t - b) \\ &+ \Pr[\mathcal{T}_2 < zq^1 | \mathcal{T}_2 < \sigma^1(b)](t - b) \end{aligned}$$

Since $zq^1 < \sigma^1(b)$, the expected utility function can be rewritten as

$$U^1(t, b, q^1) = \frac{F \circ \sigma^1(b) - F(zq^1)}{F \circ \sigma^1(b)}(q^1 + \alpha_R t - b) + \frac{F(zq^1)}{F \circ \sigma^1(b)}(t - b)$$

The optimization problem is $\max_{q^1} U^1(t, b, q^1)$. The first-order condition is

$$(1 - \alpha_R)t = q^1 - \frac{F \circ \sigma^1(b) - F(zq^1)}{zf(zq^1)} \tag{20}$$

Let $q^1(t, \sigma^1(b))$ be the resale price that solves (20). From Lemmas C.1 and C.2, it follows that

1. (20) is also sufficient.
2. There exists a unique q^1 that solves (20).

3. The resale price $q^1(t,\sigma^1(b))$ is strictly increasing in value t and bid b.

Consider the bid date and bidder 1 with value t and bid b. The expected utility function of bidder 1 is

$$\begin{aligned} U^1(t,b) = &[F\circ\sigma^1(b) - F(zq^1(t,\sigma^1(b)))](q^1+\alpha_R t - b)\\ &+ F(zq^1(t,\sigma^1(b)))(t-b)\\ &+ \int_{\sigma^1(b)}^{\bar{a}} \max\{(1-\alpha_B)t - q^1(t,\omega), 0\} f(\omega)\mathrm{d}\omega \end{aligned}$$

Using Envelope theorem and Leibniz integral rule, the first-order differential equation is

$$\frac{F\circ\sigma^1(b)}{\mathrm{D}F\circ\sigma^1(b)} = 2q^1(b) + (\alpha_R+\alpha_B-1)\sigma^1(b) - b \tag{21}$$

where $q^1(\sigma^1(b),\sigma^1(b)) = q^1(b)$.

We now show sufficiency. Suppose a pair (σ^1, q^1) solves (12). We argue that (σ^1, q^1) is an equilibrium. Consider bidder 1 with value t. Suppose he overbids to c, where $\sigma^1(c) > t$. Note that $\max\{(1-\alpha_B)t - q^1(t,\sigma^1(c)), 0\} \geq (1-\alpha_B)t - q^1(t,\sigma^1(c))$ and $q^1(\sigma^1(c),\sigma^1(c)) > q^1(t,\sigma^1(c))$. Then,

$$\begin{aligned} \mathrm{D}_c U^1(t,c) &= \mathrm{D}F\circ\sigma^1(c)[q^1(t,\sigma^1(c)) + \alpha_R t - c\\ &\quad - \max\{(1-\alpha_B)t - q^1(t,\sigma^1(c)), 0\}] - F\circ\sigma^1(c)\\ &\leq \mathrm{D}F\circ\sigma^1(c)[2q^1(t,\sigma^1(c)) + (\alpha_R+\alpha_B-1)t - c] - F\circ\sigma^1(c)\\ &< \mathrm{D}F\circ\sigma^1(c)[2q^1(\sigma^1(c),\sigma^1(c)) + (\alpha_R+\alpha_B-1)\sigma^1(c) - c]\\ &\quad - F\circ\sigma^1(c)\\ &= 0 \end{aligned}$$

Thus, overbids are not profitable for bidder 1.

Suppose he underbids to c, where $\sigma^1(c) < t$. As $q^1(\sigma^1(c),\sigma^1(c)) < q^1(t,\sigma^1(c))$, we have

$$\begin{aligned} \mathrm{D}_c U^1(t,c) &= \mathrm{D}F\circ\sigma^1(c)[q^1(t,\sigma^1(c)) + \alpha_R t - c\\ &\quad - \max\{(1-\alpha_B)t - q^1(t,\sigma^1(c)), 0\}] - F\circ\sigma^1(c)\\ &> \mathrm{D}F\circ\sigma^1(c)[q^1(c) + \alpha_R t - c\\ &\quad - \max\{(1-\alpha_B)t - q^1(c), 0\}] - F\circ\sigma^1(c) \end{aligned}$$

As $q^1(c) < (1-\alpha_B)\sigma^1(c)$ and $(1-\alpha_B)\sigma^1(c) < (1-\alpha_B)t$, we have

$q^1(c) < (1-\alpha_B)t$. This implies

$$\begin{aligned}
\mathrm{D}_c U^1(t,c) &> \mathrm{D}F \circ \sigma^1(c)[q^1(c) + \alpha_R t - c \\
&\quad - \max\{(1-\alpha_B)t - q^1(c), 0\}] - F \circ \sigma^1(c) \\
&= \mathrm{D}F \circ \sigma^1(c)[2q^1(c) + (\alpha_R + \alpha_B - 1)t - c] - F \circ \sigma^1(c) \\
&> \mathrm{D}F \circ \sigma^1(c)[2q^1(c) + (\alpha_R + \alpha_B - 1)\sigma^1(c) - c] - F \circ \sigma^1(c) \\
&= 0
\end{aligned}$$

Thus, underbids are not profitable for bidder 1. So, (σ^1, q^1) is optimal. ■

Proof of Proposition 10. Let (σ^2, q^2) be an equilibrium. We apply the process of backward induction. Without loss of generality, consider bidder 1 with a value of t. Suppose he bids b and makes a resell offer at a price of q^2.

In Lemma B.2 of Appendix B, we show that the loser always offers the object at date 2. Since bidder 1 tries to buy during resale, it must be true that he has lost the auction at date 1. This is possible if and only if $b < \mu^2(\mathcal{T}_2)$ which is equivalent to $\mathcal{T}_2 > \sigma^2(b)$. His offer gets accepted if and only if the resale price is more than the ex-post value of bidder 2, i.e., $q^2 > (1-\alpha_R)\mathcal{T}_2$, or equivalently $\mathcal{T}_2 < yq^2$, where $y = 1/(1-\alpha_R)$. On the other hand, his offer gets rejected if and only if $\mathcal{T}_2 > yq^2$. Therefore, the expected utility function of bidder 1 is

$$\begin{aligned}
U^2(t,b,q^2) &= \Pr(\mathcal{T}_2 < yq^2 | \mathcal{T}_2 > \sigma^2(b))[(1-\alpha_B)t - q^2] \\
&= \frac{F(yq^2) - F \circ \sigma^2(b)}{1 - F \circ \sigma^2(b)}[(1-\alpha_B)t - q^2]
\end{aligned}$$

The first-order condition gives

$$(1-\alpha_B)t = q^2 - \frac{F \circ \sigma^2(b) - F(yq^2)}{yf(yq^2)} \tag{22}$$

Let $q^2(t, \sigma^2(b))$ be the resale price that solves (22). From Lemmas C.1 and C.2, it follows that

1. (22) is also sufficient.
2. There exists a unique q^2 that solves (22).
3. The resale price $q^2(t, \sigma^2(b))$ is strictly increasing in value t and bid b.

Consider date 1 where a second-price auction happens. Consider bidder 1 with value t. Suppose he bids b while bidder 2 implements σ^2. He wins if and only if $\mathcal{T}_2 < \sigma^2(b)$. Whenever he wins, he receives a resale offer of $q^2(t, \mathcal{T}_2)$ from bidder 2. He accepts if and only if $q^2(t, \mathcal{T}_2) + \alpha_R t > t$, otherwise he rejects. Therefore, with probability that $\mathcal{T}_2 < \sigma^2(b)$, he incurs a utility of $\max\{q^2(t,\mathcal{T}_2)+\alpha_R t, t\} - \mu^2(\mathcal{T}_2)$ where $\mu^2(\mathcal{T}_2)$ are his payments.

Bidder 1 loses if and only if $\mathcal{T}_2 > \sigma^2(b)$. Whenever he loses, he proposes a resale offer to bidder 2. Bidder 2 accepts if and only if $q^2(t,\sigma^2(b)) > (1-\alpha_R)\mathcal{T}_2$, otherwise he rejects. Therefore, with probability that $\sigma^2(b) < \mathcal{T}_2 < yq^2(t,\sigma^2(b))$, bidder 1 gets a utility of $(1-\alpha_B)t - q^2(t,\sigma^2(b))$. Thus, the expected utility function of bidder 1 is

$$\begin{aligned}
U^2(t,b) &= \Pr(\mathcal{T}_2 < \sigma^2(b))[\max\{q^2(t,\mathcal{T}_2)+\alpha_R t,t\} - \mu^2(\mathcal{T}_2)] \\
&\quad + \Pr(\sigma^2(b) < \mathcal{T}_2 < yq^2(t,\sigma^2(b)))[(1-\alpha_B)t - q^2(t,\sigma^2(b))] \\
&= \int_0^{\sigma^2(b)} [\max\{q^2(t,\omega)+\alpha_R t,t\} - \mu^2(\omega)] f(\omega)\mathrm{d}\omega \\
&\quad + [F(yq^2(t,\sigma^2(b))) - F\circ\sigma^2(b)][(1-\alpha_B)t - q^2(t,\sigma^2(b))]
\end{aligned}$$

Using Envelope theorem and Leibniz integral rule, the first-order derivative is

$$\begin{aligned}
\mathrm{D}_b U^2(t,b) = \mathrm{D}F\circ\sigma^2(b)[&\max\{q^2(t,\sigma^2(b))+\alpha_R t,t\} - b \\
&- (1-\alpha_B)t + q^2(t,\sigma^2(b))]
\end{aligned} \tag{23}$$

For notational convenience, let $q^2(\sigma^2(b),\sigma^2(b)) = q^2(b)$. In equilibrium, $t=\sigma^2(b)$, $q^2(b) > (1-\alpha_R)\sigma^2(b)$ and $\mathrm{D}_b U^1(\sigma^2(b),b) = 0$. This gives

$$\sigma^2(b) = \frac{b - 2q^2(b)}{\alpha_R + \alpha_B - 1} \tag{24}$$

Conversely, consider a pair (σ^2, q^2) that solves (14). We show (σ^2, q^2) is an equilibrium. Consider bidder 1 with value t. Suppose he overbids to c, where $\sigma^2(c) > t$. Then, $q^2(\sigma^2(c),\sigma^2(c)) > q^2(t,\sigma^2(c))$. As $q^2(\sigma^2(c),\sigma^2(c)) > (1-\alpha_R)\sigma^2(c) > (1-\alpha_R)t$, we have $q^2(\sigma^2(c),\sigma^2(c)) + \alpha_R t > t$. Thus,

$$\begin{aligned}
\mathrm{D}_c U^2(t,c) &= \mathrm{D}F\circ\sigma^2(c)[\max\{q^2(t,\sigma^2(c))+\alpha_R t,t\} - c - (1-\alpha_B)t \\
&\quad + q^2(t,\sigma^2(c))] \\
&< \mathrm{D}F\circ\sigma^2(c)[\max\{q^2(c)+\alpha_R t,t\} - c - (1-\alpha_B)t + q^2(c)] \\
&= \mathrm{D}F\circ\sigma^2(c)[q^2(c)+\alpha_R t - c - (1-\alpha_B)t + q^2(c)] \\
&< \mathrm{D}F\circ\sigma^2(c)[2q^2(c) + (\alpha_R+\alpha_B-1)\sigma^2(c) - c] \\
&= 0
\end{aligned}$$

Therefore, overbids are not profitable for bidder 1.

Now, suppose bidder 1 underbids to c, where $\sigma^2(c) < t$. Then $q^2(\sigma^2(c),\sigma^2(c)) < q^2(t,\sigma^2(c))$. As

$$\max\{q^2(t,\sigma^2(c))+\alpha_R t,t\} \geq q^2(t,\sigma^2(c)) + \alpha_R t,$$

we have

$$\begin{aligned}
\mathrm{D}_c U^2(t,c) &= \mathrm{D}F \circ \sigma^2(c)[\max\{q^2(t,\sigma^2(c)) + \alpha_R t, t\} - c - (1-\alpha_B)t \\
&\quad + q^2(t,\sigma^2(c))] \\
&\geq \mathrm{D}F \circ \sigma^2(c)[2q^2(t,\sigma^2(c)) + (\alpha_R + \alpha_B - 1)t - c] \\
&> \mathrm{D}F \circ \sigma^2(c)[2q^2(c) + (\alpha_R + \alpha_B - 1)\sigma^2(c) - c] \\
&= 0
\end{aligned}$$

Thus, underbids are not profitable for bidder 1. Hence, (σ^2, q^2) is an equilibrium. ■

Proof of Proposition 11. To contradict, let $\pi^1 \geq \sigma^1$ around a neighborhood of 0. Then, from (5) and (12), we have

$$\begin{aligned}
\frac{F \circ \sigma^1(b)}{\mathrm{D}F \circ \sigma^1(b)} &= 2q^1(b) + (\alpha_R + \alpha_B - 1)\sigma^1(b) - b \\
&< 2(1-\alpha_B)\sigma^1(b) + (\alpha_R + \alpha_B - 1)\sigma^1(b) - b \\
&= (1 + \alpha_R - \alpha_B)\sigma^1(b) - b \\
&\leq (1 + \alpha_R - \alpha_B)\pi^1(b) - b \\
&= \frac{F \circ \pi^1(b)}{\mathrm{D}F \circ \pi^1(b)}
\end{aligned}$$

This implies

$$\mathrm{D}\left[\frac{F \circ \sigma^1(b)}{F \circ \pi^1(b)}\right] > 0$$

As $F(0) > 0$ and $\pi^1(0) = \sigma^1(0) = 0$, we have $\sigma^1 > \pi^1$ around a neighborhood of 0 which is a contradiction.

Now, suppose that there exists $b^* > 0$ so that $\sigma^1(b^*) = \pi^1(b^*)$ and $\sigma^1(b) > \pi^1(b)$ for every $b \in (0, b^*]$. Then, we have

$$\begin{aligned}
\mathrm{D}\sigma^1(b^*) &= \frac{F \circ \sigma^1(b^*)}{f \circ \sigma^1(b^*)} \frac{1}{2q^1(b^*) + (\alpha_R + \alpha_B - 1)\sigma^1(b^*) - b^*} \\
&> \frac{F \circ \sigma^1(b^*)}{f \circ \sigma^1(b^*)} \frac{1}{2(1-\alpha_B)\sigma^1(b^*) + (\alpha_R + \alpha_B - 1)\sigma^1(b^*) - b^*} \\
&= \frac{F \circ \sigma^1(b^*)}{f \circ \sigma^1(b^*)} \frac{1}{(1 + \alpha_R - \alpha_B)\sigma^1(b^*) - b^*} \\
&= \frac{F \circ \pi^1(b^*)}{f \circ \pi^1(b^*)} \frac{1}{(1 + \alpha_R - \alpha_B)\pi^1(b^*) - b^*} \\
&= \mathrm{D}\pi^1(b^*)
\end{aligned}$$

Thus, there exist $\delta > 0$ so that $\sigma^1(b^* - \delta) < \pi^1(b^* - \delta)$, which is a contradiction. ■

Proof of Proposition 13. Note that $q^2(b) > (1-\alpha_R)\sigma^2(b)$. From (14), we have

$$\begin{aligned}\sigma^2(b) &= \frac{b - 2q^2(b)}{\alpha_R + \alpha_B - 1} \\ &< \frac{b - 2(1-\alpha_R)\sigma^2(b)}{\alpha_R + \alpha_B - 1}\end{aligned}$$

which equals

$$\begin{aligned}\sigma^2(b) &< \frac{b}{1 + \alpha_B - \alpha_R} \\ &= \pi^2(b)\end{aligned}$$

where the last equality follows from (10). ■

B Appendix: Direction of resale

Lemma B.1. *Under the monopoly rule, incomplete information and the first-price auction, whosoever wins offer the object at the resale date.*

Proof. Since bidders are symmetric, without loss of generality, consider bidder 1 with value t_1. Suppose he wins with a bid of b. Since bid functions are symmetric, it must be the case that $t_1 > t_2$, which is equivalent to $(1-\alpha_R)t_1 > (1-\alpha_R)t_2$. From Assumption 1, we have $(1-\alpha_B)t_2 > (1-\alpha_R)t_2$. Therefore, with positive probability, we have $(1-\alpha_R)t_1 < (1-\alpha_B)t_2$, where $(1-\alpha_R)t_1$ is the ex-post value of bidder 1 (reseller) at the resale date while $(1-\alpha_B)t_2$ is the ex-post value of bidder 2 (buyer) at the resale date. Thus, there are expected potential profits if bidder 1 offers the object to bidder 2 at the resale date. ■

Lemma B.2. *Under the monopsony rule and incomplete information, whosoever loses offer the object for resale under the monopsony rule.*

Proof. Without loss of generality, consider bidder 1 with value t_1. Suppose he loses with a bid of b. Then, $t_1 < t_2$ which is equivalent to $(1-\alpha_B)t_1 < (1-\alpha_B)t_2$. From Assumption 1, $(1-\alpha_B)t_2 > (1-\alpha_R)t_2$. This implies, with a positive probability, we have $(1-\alpha_B)t_1 > (1-\alpha_R)t_2$, where $(1-\alpha_B)t_1$ is the ex-post value of bidder 1 at date 2 while $(1-\alpha_R)t_2$ is the ex-post value of bidder 2 at date 2. Therefore, there are expected potential gains from trade. ■

C Appendix: Technical lemmas

Lemma C.1. *Let Assumption 3 be true. The expression*

$$\frac{f(xq)}{F(a) - F(xq)}$$

is non-decreasing in q for every $x \in \Re_+$ and $a \in (0, \bar{a})$.

Proof. Pick $q_1, q_2 \in \Re_+$ so that $q_1 > q_2$. We show

$$f(xq_1)[F(a) - F(xq_2)] - f(xq_2)[F(a) - F(xq_1)] \geq 0 \tag{25}$$

From Assumption 2, we have $f(xq_1)[1 - F(xq_2)] \geq f(xq_2)[1 - F(xq_1)]$ as $x > 0$.

If $f(xq_1) > f(xq_2)$, the the result follows as $F(xq_1) > F(xq_2)$. If $f(xq_1) \leq f(xq_2)$, the derivative of left hand side of (25) with respect to $F(a)$ is $f(xq_1) - f(xq_2) \leq 0$. Thus, the result holds. ■

Lemma C.2. *Let Assumption 3 be true. Then, (20) and (22) are sufficient.*

Proof. We show that (22) is sufficient. On similar lines, one can show that (20) is sufficient. The first-order derivative of (22) gives

$$\begin{aligned}
\mathrm{D}_{q^2}U^1(t, b, q^2) &= \frac{1}{1 - F \circ \sigma^2(b)}\Big\{yf(yq^2)[(1 - \alpha_B)t - q^2] + \\
&\quad [F \circ \sigma^2(b) - F(yq^2)]\Big\} \\
&= \frac{yf(yq^2)}{1 - F \circ \sigma^2(b)}\left\{[(1 - \alpha_B)t - q^2] + \frac{F \circ \sigma^2(b) - F(yq^2)}{yf(yq^2)}\right\}
\end{aligned}$$

The second-order derivative is

$$\begin{aligned}
\mathrm{D}^2_{q^2}U^1(t, b, q^2) &= \frac{yf(yq^2)}{1 - F \circ \sigma^2(b)}\left\{-1 + \mathrm{D}_{q^2}\frac{F \circ \sigma^2(b) - F(yq^2)}{yf(yq^2)}\right\} \\
&\quad + \frac{y^2\mathrm{D}f(yq^2)}{1 - F \circ \sigma^2(b)}\left\{[(1 - \alpha_B)t - q^2] + \frac{F \circ \sigma^2(b) - F(yq^2)}{yf(yq^2)}\right\} \\
&= \frac{yf(yq^2)}{1 - F \circ \sigma^2(b)}\left\{-1 + \mathrm{D}_{q^2}\frac{F \circ \sigma^2(b) - F(yq^2)}{yf(yq^2)}\right\} \\
&< 0
\end{aligned}$$

where the last inequality follows from Lemma B.1. ■

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