Ashoka University
Economics Discussion Paper 120

Difference-in-Differences with a Misclassified Treatment

August 2024

Akanksha Negi, Monash University
Digvijay S. Negi, Ashoka University

https://ashoka.edu.in/economics-discussionpapers

Plot No. 2, Rajiv Gandhi Education City,
National Capital Region,
P.O.Rai, Sonepat, Haryana 131029

Difference-in-Differences with a Misclassified Treatment

Akanksha Negi[†] and Digvijay S. Negi[‡]

Abstract

This paper studies identification and estimation of the average treatment effect of a latent treated subpopulation in difference-in-difference designs when the observed treatment is differentially (or endogenously) mismeasured for the truth. Common examples include misreporting and mistargeting. We propose a two-step estimator which corrects for the empirically common phenomenon of one-sided misclassification in the treatment status. The solution uses a single exclusion restriction embedded in a partial observability probit to point-identify the latent parameter. We demonstrate the method by revisiting two large-scale national programs in India; one where pension benefits are under-reported and second where the program is mistargeted.

JEL Classification Codes: C21, C23, C51

Keywords: Difference-in-differences, Misclassification, Heterogeneous treatment effects, Panel data, Repeated cross-sections

[†]Department of Econometrics and Business Statistics, Monash University, Australia. Email: akanksha.negi@monash.edu.

[‡]Ashoka University, Rajiv Gandhi Education City, Sonepat, Haryana, India. Email: digvijay.negi@ashoka.edu.in .

[*] We would like to thank David Kaplan, Denni Tommasi, Kaspar Wuthrich, Daniel Millimet, Christian Cox, and seminar participants at EWMES 2021, ESAM 2022, Duke Microeconometrics Class of 2020 & 2021 Conference, Microeconometrics and Policy Evaluation Session at BSE Summer Forum, Department of Econometrics and Business Statistics at Monash, and Department of Economics at UTS, University of Melbourne, University of Queensland, and Monash University, for comments and suggestions.

1 Introduction

The degree of measurement error in economic data and its influence on parameter estimates have been of much interest to econometricians. One case in particular is measurement error in a binary variable which is also known as misclassification. Unlike classical measurement error, where the error is uncorrelated with the truth, misclassification is necessarily non-classical since the error is negatively correlated with the true value of the variable itself (Aigner (1973); Bound et al. (2001)).[1] This is especially relevant in the program evaluation setting, of which difference-in-differences (DID) is a workhorse empirical strategy, because the key regressor of interest, the *treatment*, is often binary in nature.[2]

In this paper, we discuss identification and estimation of the average treatment effect on the treated for an unobserved treatment D^* (latent ATT) in a DID setting, when the binary variable that classifies individuals into treatment or control, D, is mismeasured for D^*. Participation in programs or interventions is particularly prone to mismeasurement (Bruckmeier et al. (2021); Martinelli and Parker (2009); Cornia and Stewart (1993)). For instance, Meyer et al. (2015) document how misreporting of program receipt and conditional transfers from the government appear to be the biggest threat to household survey data quality for policy evaluation. Another relatively understudied problem is mistargeting of targeted interventions.[3] Mistargeting errors arise from the misclassification of true program beneficiaries and result in the wrongful inclusion or exclusion of households from receiving program benefits (Cornia and Stewart (1993)).

Given that DID is often applied to data from nationally representative surveys (Groen

[1]While it is commonly understood that a linear instrumental variables strategy can be used for dealing with classical measurement error, it is severely biased when the mismeasured variable is binary (Loewenstein and Spletzer (1997); Black et al. (2000); Kane et al. (1999); DiTraglia and Garcia-Jimeno (2019)).

[2]Evidence from the literature suggest that ignoring even a small amount of misclassification can have major repercussions for the estimated treatment effects Millimet (2011); Kreider (2010).

[3]Cameron and Shah (2014) report an extreme case from Indonesia where mistargeting of a cash transfer program led to destruction of trust and social capital and an increase in criminal behavior.

and Polivka (2008); Buchmueller et al. (2011); Botosaru and Gutierrez (2018)) and is routinely used for assessing the impact/performance of targeted programs, it becomes important to study the effects of a misclassified treatment in the DID setting. We distinguish between two important examples of misclassification in the treatment status. The first represents a misreporting-type error mechanism where the treatment *received* is different from what is *recorded/observed*. Examples include self-reports by individuals (Celhay et al. (2024)), documentation errors (nonrandom data-entries), and even imputation-led errors (as discussed in Sjoquist and Winters (2015)). The second characterizes a mistargeting-type error phenomenon. This covers cases where treatment originally *intended* under a program or policy is different from what's *received/observed*. Targeting errors often emerge due to imperfect information about eligibility, corruption, political connections, and even elite capture (Cornia and Stewart (1993); Pande (2007); Besley et al. (2012); Alatas et al. (2012); Niehaus and Sukhtankar (2013); Panda (2015)).

Importantly, in both cases, we have a misclassified proxy D for the latent treatment D^*. We show that for misreporting-type errors, DID using observed treatment is biased for the latent ATT. In this case, latent ATT is interpreted as the average treatment effect for those who actually received the program. The bias in the DID estimand takes a simple attenuation form if misreporting is non-differential (or exogenous); i.e. measurement error is independent of outcomes once we condition on D^* but is harder to 'sign' if misreporting is differential (or endogenous) i.e, measurement error is not independent of the outcome even after conditioning on D^*. In contrast, with mistargeting-type errors, recovering the ATT of actual receipt is not feasible and our focus is rather on the average effect of being intended to be treated under the program. We refer to this as ATITT, which is how the latent parameter is interpreted in this context. We argue that this is a more interesting parameter and one that be salvaged (causally identified) in a mistargeting setting.

Our general solution is derived under the assumption of conditional parallel trends with respect to D^*. We use flexible linear specifications of the potential outcome means to arrive at parametric DID regressions for two-period panel and repeated cross-section settings that allow for heterogeneity in covariates. Our approach allows for differential misclassification in D that is predominantly one-sided. This is not necessarily a drawback since misclassification may often be driven, in a large part, by either errors of exclusion or inclusion depending on the empirical context. Such one-sided errors are formulated using a partial observability probit (POP). The proposed estimator then uses the predicted probabilities of D^* from the first-step estimation of the POP equation in the second-stage DID regressions to estimate the latent ATT. The resulting two-step DID estimators (for both panel and repeated cross-sections) are shown to be consistent and asymptotically normal.

Our solution extends Nguimkeu et al. (2019) (NDT hereafter), who study endogenous participation and endogenous misreporting of social programs, to a DID setup. In their case, two exclusion restrictions are required for identifying the parameter of interest; one corresponding to the true treatment equation and another for the misclassification equation. In our case, having a time dimension allows us to address any time-invariant endogeneity in D^* without requiring an exclusion restriction. We view this as an improvement over NDT. Consequently, the latent ATT is identified with a single exclusion restriction that affects the misclassification probability but does not affect the true treatment or outcome equation.

Finally, we illustrate the method with two large-scale transfer programs in India. The first is the Indira Gandhi National Old Age Pension Scheme, which is the federal government's social security program for old individuals. It is well-known that the benefits received from government programs are under-reported (Stecklov et al. (2018)). The second, known as the Targeted Public Distribution System, is an in-kind transfer program which distributes subsidized food grains to poor eligible households. A key challenge here is that the program suffers from high exclusion errors (Dutta and Ramaswami (2001); Swaminathan and Misra (2001)). We use the proposed estimator and provide estimates of ATT and ATITT after correcting for under-reporting and one-sided mistargeting, respectively.

To the best of our knowledge, this is the first paper to discuss treatment misclassification in a DID framework. Concurrently, Denteh and Kédagni (2022) study this problem and propose bounds for the ATT under both-sided errors. We see their analysis as complementary to our objective of point identification in the presence of one-sided error, which is still the dominant scenario in applications. This paper contributes generally to the literature dealing with binary misclassified regressors (Aigner (1973), Bollinger (1996), Mahajan (2006), Lewbel (2007), Frazis and Loewenstein (2003), Meyer and Mittag (2017), Haider and Stephens Jr (2020), Battistin and Sianesi (2011), DiTraglia and Garcia-Jimeno (2019)) and specifically to the sparse literature dealing with differential misclassification of an endogenous treatment. Some recent works include partial and point identification of the local average treatment effect (Ura (2018), Tommasi and Zhang (2024), Yanagi (2019), Calvi et al. (2021)) and point identification of marginal treatment effects (Acerenza et al. (2021)).

There is also a vast literature studying DID methods. We align more with the strand that looks at the canonical DID setup such as Abadie (2005), De Chaisemartin and d'Haultfoeuille (2018), Botosaru and Gutierrez (2018), and Sant'Anna and Zhao (2020). In particular, our paper is related to Botosaru and Gutierrez (2018) who study DID with repeated cross-sections when the treatment status is missing in one of the two periods. Their solution relies on observing proxies in both periods for identifying the ATT. A key assumption is that the proxies must not affect the change in outcomes after conditioning on the true treatment status, making it similar to a non-differential error assumption. In contrast, we allow for measurement error to be differential. Moreover, our method doesn't require observing a variable (proxy) in both periods and works in both panels and repeated cross-sections.

The rest of the paper is organized as follows. Section 2 describes the theory of misclassification in a DID framework along with the proposed solution. Section 3 discusses application of the theory to two cases of misclassification; misreporting and mistargeting. Section 4 presents empirical results from two separate transfer programs in India. Section 5 concludes.

2 DID with Misclassification

2.1 Design

Consider a setting where there are two time periods, $t = 0$ and $t = 1$. Let D_t^* denote a *latent* (i.e. unobserved) binary treatment of interest such that $D_0^* = 0$ for everyone and $D_1^* \equiv D^*$. Let D be a misclassified proxy for D^* such that some units are observed as being treated between periods 0 and 1.

Let $Y_t(D^*)$ denote the potential outcome at time period t. Then,

$$Y_t = Y_t(0) \cdot (1 - D^*) + Y_t(1) \cdot D^* \quad \text{for each} \;\; t = 0, 1. \tag{1}$$

Our parameter of interest, $\tau \equiv \mathbb{E}[Y_1(1) - Y_1(0)|D^* = 1]$, is defined as the ATT for the latent subpopulation, $D^* = 1$.

Let $X = (X_1, X_2, \ldots, X_k)$ be a vector of pre-treatment covariates. For notation brevity, we will use $R = (1, X)$ to be the vector which includes an intercept. Also, let Δ be the first-differencing (FD) operator such that for any random variable, W, $\Delta W \equiv W_1 - W_0$ represents the change between time periods, 0 and 1.

Assumption 1 (**Conditional parallel trends**). $\mathbb{E}[\Delta Y(0)|X, D^*] = \mathbb{E}[\Delta Y(0)|X]$.

Assumption 2 (**Overlap**). $\mathbb{P}(D^* = 1) > 0$ *and* $\mathbb{P}(D^* = 1|X) < 1$.

Assumption 3 (**No-anticipation**). $\mathbb{E}[Y_0(1) - Y_0(0)|X, D^* = 1] = 0$.

Assumption (1) imposes that the average trend in the untreated potential outcome of the two groups would have evolved in parallel between period 0 and 1 conditional on X. This allows for covariate specific time trends in the untreated potential outcome means.[4] Assumption 2 is the overlap condition which is required for identifying the ATT for each subpopulation of X and Assumption 3 states that there are no anticipatory effects of the treatment in period 0, conditional on X. All three assumptions are quite standard in the conditional DID literature.

[4]Other papers that use conditional parallel trends include Abadie (2005), Callaway and Sant'Anna (2020), Sant'Anna and Zhao (2020), and Wooldridge (2021).

Remark 1. *As is well known, under assumptions 1-3, if D^* were perfectly observed, one could identify τ using conditional DID methods.*

2.2 Linear DID

Using (1) and the definition of Δ, we can write $Y_1 = Y_0(0) + \Delta Y(0) + D^* \cdot (Y_1(1) - Y_1(0))$ and $Y_0 = Y_0(0) + D^* \cdot (Y_0(1) - Y_0(0))$. The conditional means of Y_t given D^* are then given as

$$\begin{aligned} \mathbb{E}[Y_0|X, D^*] &= \mathbb{E}[Y_0(0)|X, D^*] \quad \text{(using no-anticipation) and} \\ \mathbb{E}[Y_1|X, D^*] &= \mathbb{E}[Y_0(0)|X, D^*] + \mathbb{E}[\Delta Y(0)|X, D^*] + D^* \cdot \tau(X). \end{aligned} \tag{2}$$

The second equality follows from the simple characterization that $\mathbb{E}[Y_1(1)-Y_1(0)|X, D^*] = D^* \cdot \mathbb{E}[Y_1(1) - Y_1(0)|X, D^* = 1] + (1 - D^*) \cdot \mathbb{E}[Y_1(1) - Y_1(0)|X, D^* = 0]$ which implies that $D^* \cdot \mathbb{E}[Y_1(1) - Y_1(0)|X, D^*] = D^* \cdot \mathbb{E}[Y_1(1) - Y_1(0)|X, D^* = 1] \equiv D^* \cdot \tau(X)$. Let

$$\mathbb{E}[Y_0(0)|X, D^*] = \eta_{00} + \eta_{01} D^* + \mathring{X}\eta_{02} + D^*\mathring{X}\eta_{03} \tag{3}$$

$$\mathbb{E}[\Delta Y(0)|X] = \delta_{01} + \mathring{X}\delta_{11} \text{ and} \tag{4}$$

$$\tau(X) = \tau + \mathring{X}\kappa \tag{5}$$

where $\mathring{X} = X - \mathbb{E}(X|D^* = 1)$. Define $\mathring{R} \equiv (1, \mathring{X})$ and $W^* \equiv D^*\mathring{R}$. Note that the above framework allows the conditional means to be flexible in X and D^* and also allows heterogeneity in the ATT based on covariates.

Two-period panel Substituting (3), (4), (5) in (2) and taking first differences, we can write the equation in error form as

$$\Delta Y = \mathring{R}\delta + W^*\theta + \Delta\xi \text{ such that } \mathbb{E}[\Delta\xi|R, D^*] = 0. \tag{6}$$

Here $\delta = (\delta_{01}, \delta'_{11})'$, and $\theta = (\tau, \kappa')'$.

Repeated cross sections Let T is a binary indicator for the post-treatment period. Then $Y = Y_1 \cdot T + Y_0 \cdot (1 - T)$. Using (2)-(5), we can write Y in error form as

$$Y = \mathring{R}\eta_1 + \mathring{W}\eta_2 + T \cdot \mathring{R}\delta + T \cdot \mathring{W}\theta + \xi, \text{ such that } \mathbb{E}[\xi|R, D^*, T] = 0 \tag{7}$$

where $\eta_1 = (\eta_{00}, \eta_{02}')'$, $\eta_2 = (\eta_{01}, \eta_{03}')'$, $\xi = T \cdot \xi_1 + (1-T)\xi_0$ where ξ_t is the error associated with Y_t, $t = 0, 1$.

2.3 Identification with Partial Observability Probit

Assume that

$$D = D^* \cdot S \tag{8}$$

where S is a binary indicator for correct classification.[5] This is a one-sided error formulation since $D = 1$ implies $D^* = 1$ but can never imply $D^* = 0$. On the other hand, $D = 0$ can either imply $D^* = 1$ or $D^* = 0$. In other words, errors of exclusion are permitted whereas errors of inclusion are ruled out.

Assume that $D^* = \mathbb{1}\{R\gamma + U \geq 0\}$ and $S = \mathbb{1}\{Z\alpha + V \geq 0\}$ where R is the vector of exogenous covariates that predict D^* and Z is the $1 \times p$ vector of instruments that predict S. γ and α are parameter vectors of size $k + 1$ and p, respectively, and U and V are the two latent errors.

Let the conditional distribution of $(-U, -V)$ be bivariate normal with a CDF given by $F_{U,V}(\cdot, \cdot; \rho)$ where ρ is the correlation coefficient. Then

$$\mathbb{P}(D = 1|R, Z) = \mathbb{P}(-U \leq R\gamma, -V \leq Z\alpha) = F_{U,V}(R\gamma, Z\alpha; \rho). \tag{9}$$

defines a partial observability probit (POP). Identification of parameters $(\alpha, \gamma, \rho)_{(k+p+2)}$ in this model requires atleast one exogenous variable in Z that is excluded from R. In

[5] As discussed in Acerenza et al. (2021), $D = D^* \cdot S + (1 - D^*) \cdot (1 - S)$ is the general error formulation which allows both kinds of classification errors. This is easily derived from the more common additively separable form, $D = D^* + \varepsilon$.

addition, the exogenous variables must take on atleast as many distinct values as the number of unknown parameters. This is easily achieved by including continuous variables in X and Z. These conditions are needed for local identification of the POP parameters (see Poirier (1980) for a longer discussion). The next assumption formalizes the relationship between the outcome, participation, and misclassification equation errors.

Assumption 4. *Assume that a) The error term $\Delta\xi$ is independent of R, Z, with variance σ^2; and the error terms (U, V) are independent of all covariates R, Z and have unit variances. The correlations for the pairs $(\Delta\xi, V)$ and (U, V) are denoted ψ_v and ρ, respectively. b) The error terms $(\Delta\xi, U, V)$, follow a trivariate normal distribution, conditional on all covariates (R, Z) i.e.*

$$(\Delta\xi, U, V)'|R, Z \sim N(0, \Sigma) \text{ where } \Sigma = \begin{pmatrix} \sigma^2 & 0 & \psi_v\sigma \\ 0 & 1 & \rho \\ \psi_v\sigma & \rho & 1 \end{pmatrix}.$$

In the case of repeated cross sections, simply replace $\Delta\xi$ with the pooled error ξ, where the conditioning set includes T.

We see that the correlation between the outcome error and U is zero. This is because D^* is not allowed to be correlated with time-varying unobservables in a DID. This is a salient difference between the current analysis and NDT. The latter require atleast two exclusion restrictions; one for the endogeneity of D^* and the other for endogenous S. In contrast, we require only one exclusion restriction since time-invariant endogeneity of D^* is allowed under the DID design. V and the outcome error are allowed to be correlated because of differential misclassification. Note that joint normality of the errors is not necessary for the procedure to work (see Nguimkeu et al. (2019)).

To put it in familiar terms, the POP model and assumption 4 together contain the following identification assumptions: i)$\mathbb{P}(S = 1|R, Z) \neq \mathbb{P}(S = 1|R)$, ii)$\mathbb{E}(\Delta\xi|R, Z, D^*) = 0$, and iii) $\mathbb{E}(\Delta\xi|R, Z, D^*, D) \neq \mathbb{E}(\Delta\xi|R, Z, D^*)$. Part i) is the instrument relevance assumption that imposes that misclassification probabilities depend on the instrument, part ii) implies that the instrument and latent treatment are both exogenous to the outcome error, and part iii) states that misclassification is differential because outcomes are not independent of the measurement error even if we condition on the latent D^*.

2.4 Two-step Estimator

Assume that the researcher has access to a random sample. Formally,

Assumption 5 (**Random sampling**). *Assume that the data are either independent and identically distributed from i) Panel: $\{(Y_{it}, X_i, D_i);\ i = 1, 2, \dots, N\}$, for $t = 0, 1$; or ii) Repeated cross section: $\{(Y_i, D_i, T_i, X_i); i = 1, 2, \dots, N\}$ where $\lambda \equiv \mathbb{P}(T = 1) \in (0, 1)$ and $(Y, D, T, X) \in \mathbb{R} \times \{0, 1\} \times \{0, 1\} \times \mathbb{R}^k$.*

We propose a two-step procedure where the first-step involves estimating the POP parameters by maximizing the log-likelihood function

$$\max_{(\gamma,\alpha,\rho)} \sum_{i=1}^{N} D_i \log[F_{U,V}(R_i\gamma, Z_i\alpha; \rho)] + (1 - D_i)\log[1 - F_{U,V}(R_i\gamma, Z_i\alpha; \rho)]. \tag{10}$$

First-step: Use $\hat{\alpha}$ from equation (10) to obtain the predicted probabilities $\hat{D}_i^* = \Phi(R_i\hat{\gamma})$.[6]

Define, $\hat{R}^* \equiv (1, \hat{X}^*)$, $\hat{X}^* = X - \hat{\bar{X}}_1^*$ and $\hat{\bar{X}}_1^* = \frac{1}{\hat{N}^*}\sum_{i=1}^{N} \hat{D}_i^* \cdot X_i$ with $\hat{N}^* = \sum_{i=1}^{N} \hat{D}_i^*$ being the sum of predicted probabilities. Finally, $\hat{W}^* \equiv \hat{D}^*\hat{R}^*$.

Second-step: Use the predicted probabilities from the first-step in the regression of

$$\Delta Y_i \text{ on } \hat{R}_i^*, \hat{W}_i^*, \ \ i = 1, \dots, N. \tag{11}$$

The coefficient on $\hat{W}_i^*$ gives us the proposed two-step FD estimator $\hat{\theta}_{\text{FD}}^{\text{2S}}$.

For repeated cross sections, the two-step POLS estimator $\hat{\theta}_{\text{POLS}}^{\text{2S}}$ is obtained as the coefficient on $T_i\hat{W}_i^*$ from the second step regression,

$$Y_i \text{ on } \hat{R}_i^*, \hat{W}_i^*, T_i\hat{R}_i^*, T_i\hat{W}_i^*, \ \ i = 1, \dots, N. \tag{12}$$

Both $\hat{\theta}_{\text{FD}}^{\text{2S}}$ and $\hat{\theta}_{\text{POLS}}^{\text{2S}}$ are consistent and asymptotically normal. Formally,

[6]Implementation of first-step can be achieved using the `biprobit` command in Stata.

Theorem 1 (Asymptotic distribution of the two-step estimator)**.** *Under assumptions 1, 2, 4, 5, and full rank of the* $\mathbb{E}(\mathring{R}'\mathring{R})$ *matrix*

$$\sqrt{N}(\hat{\tau}^{2S}_{FD} - \tau) \xrightarrow{d} N(0, \Omega_{FD}) \quad \textit{and} \quad \sqrt{N}(\hat{\tau}^{2S}_{POLS} - \tau) \xrightarrow{d} N(0, \Omega_{POLS})$$

$$\Omega_{FD} = \mathbb{E}(\mathring{R}_i|D^*_i = 1) \cdot \mathbb{A}var[\sqrt{N}(\hat{\theta}^{2S}_{FD} - \theta)] \cdot \mathbb{E}(\mathring{R}_i|D^*_i = 1)'$$

$$\Omega_{POLS} = \mathbb{E}(\mathring{R}_i|D^*_i = 1) \cdot \mathbb{A}var[\sqrt{N}(\hat{\theta}^{2S}_{POLS} - \theta)] \cdot \mathbb{E}(\mathring{R}_i|D^*_i = 1)'$$

where $\mathbb{A}var[\sqrt{N}(\hat{\theta}^{2S}_{FD} - \theta)] = \Omega_\Gamma^{-1}\Omega_\psi\Omega_\Gamma^{-1}$ *and* $\mathbb{A}var[\sqrt{N}(\hat{\theta}^{2S}_{POLS} - \theta)] = \Omega_\Gamma^{-1}\left(\frac{\Omega_{\psi_\pi}}{\lambda} + \frac{\Omega_{\psi_\eta}}{(1-\lambda)}\right)\Omega_\Gamma^{-1}$.

$$\Omega_\Gamma = \mathbb{E}[\Gamma_i\Gamma_i'], \Gamma_i = \Phi(R_i\gamma)\mathring{R}_i' - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i]\mathbb{E}[\mathring{R}_i'\mathring{R}_i]^{-1}\mathring{R}_i'$$

$$\Omega_\psi = \mathbb{E}[\psi_i\psi_i'], \psi_i = \Gamma_i\Big\{\mathring{R}_i\delta + (W^*_i - \Pi^*_i(\gamma))\theta + \Delta\xi_i + \Phi(R_i\gamma)(\hat{\bar{X}}^*_1 - \mu_1)\kappa - \phi(R_i\gamma)R_i(\hat{\gamma} - \gamma)\mathring{R}_i\theta\Big\}$$

$$\Omega_{\psi_\pi} = \mathbb{E}[\psi_{i\pi}\psi_{i\pi}'], \psi_{i\pi} = \Gamma_i\Big\{\mathring{R}_i\pi_1 + (W^*_i - \Pi^*_i(\gamma))\pi_2 + \xi_{i1} + \Phi(R_i\gamma)(\hat{\bar{X}}^*_1 - \mu_1)(\eta_{03} + \kappa)$$
$$- \phi(R_i\gamma)R_i(\hat{\gamma} - \gamma)\mathring{R}_i(\eta_{01} + \tau)\Big\}$$

$$\Omega_{\psi_\eta} = \mathbb{E}[\psi_{i\eta}\psi_{i\eta}'], \psi_{i\eta} = \Gamma_i\Big\{\mathring{R}_i\eta_1 + (W^*_i - \Pi^*_i(\gamma))\eta_2 + \xi_{i1} + \Phi(R_i\gamma)(\hat{\bar{X}}^*_1 - \mu_1)\eta_{03} - \phi(R_i\gamma)R_i(\hat{\gamma} - \gamma)\mathring{R}_i\eta_{01}\Big\}$$

Even though our focus is on the unconditional ATT, the framework and asymptotic theory can be used for identification and estimation of the conditional estimand $\tau(X)$.

3 Examples

3.1 Misreporting

In a typical misreporting example, D^* is the treatment *received* whereas D documents its *reported* status. The latter may be contaminated on account of false claims of receipt or non-receipt by individuals. In this case, τ is interpreted as the average treatment effect for those who truly received the treatment. Define $q_d = \mathbb{P}(D^* = d|D = d)$ as the probability of correctly reporting treatment level $d = 0, 1$. Then, the one-sided formulation in equation (8) implies that $q_1 = 1$.

Proposition 1 (Bias under non-differential misreporting)**.** *If misreporting is non-differential* *i.e.* $\mathbb{E}[\Delta\xi|D, D^*] = \mathbb{E}[\Delta\xi|D^*]$, *then*

$$\mathbb{E}[\Delta Y|D = 1] - \mathbb{E}[\Delta Y|D = 0] = \tau \cdot q_0 \tag{13}$$

under 'no-anticipation' and 'parallel trends' in D^*.

The above proposition states that if misreporting in D is non-differential or exogenous, i.e. that mismeasured treatment contains no information on mean outcome once we condition on the true receipt, the bias in the unconditional DID estimand has the familiar attenuation form (seen most notably in Aigner (1973), Lewbel (2007), and Battistin and Sianesi (2011)). This simpler bias expression also suggests that non-differential misreporting will not cause parallel trends to be violated in D. However, this ceases to be true in the case of differential misreporting.

3.1.1 Bias in Linear DID Estimators

If misclassification is differential, the asymptotic bias is harder to 'sign'. Define $\hat{\theta}_{\text{FD}}$ and $\hat{\theta}_{\text{POLS}}$ as the estimators of θ which replace D in place of D^* in equations (6) and (7), respectively. Then

Theorem 2 (Bias under differential misreporting)**.** *Define* $\dot{R}_i = (1, \dot{X}_i)$ *where* $\dot{X}_i = X_i - \mathbb{E}(X_i|D_i = 1)$ *and* $\dot{W}_i = D_i\dot{R}_i$. *Then, given assumptions 1, 2, 3, 4, and 5, and* $\mathbb{E}[\dot{R}_i'\dot{R}_i]$ *being full rank*

$$\text{plim}(\hat{\tau}_{\text{FD}}) - \tau = \mathbb{E}[\dot{R}_i Q^{-1}(A\delta + B\theta + C)|D_i = 1]$$

$$\text{plim}(\hat{\tau}_{\text{POLS}}) - \tau = \mathbb{E}[\dot{R}_i \left\{Q_1^{-1}(A_1\pi_1 + B\pi_2 + C_1) - Q_0^{-1}(A_1\eta_1 + B\eta_2 + C_0)\right\}|D_i = 1]$$

where $Q = \mathbb{E}(\dot{W}_i'\dot{W}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)\mathbb{E}(\dot{R}_i'\dot{R}_i)^{-1}\mathbb{E}(\dot{R}_i'\dot{W}_i)$, $A = \mathbb{E}(\dot{W}_i'\mathring{R}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}(\dot{R}_i'\mathring{R}_i)$, $B = \mathbb{E}[\dot{W}_i'(W_i^* - \dot{W}_i)] - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}[\dot{R}_i'(W_i^* - \dot{W}_i)$ *and* $C = \sigma\psi_v\mathbb{E}\left[\dot{R}_i'\phi(-Z_i\alpha)\Phi\left(\frac{R_i\gamma - \rho Z_i\alpha}{\sqrt{1-\rho^2}}\right)\right]$. *Note that* Q_t, A_t, B_t *and* C_t *for* $\hat{\tau}_{\text{POLS}}$ *are defined analogously for the* $T = 0, 1$ *populations.*

As we can see from the above theorem, with differential misclassification in D, the asymptotic bias in the FD and POLS estimators does not have a simple attenuation form.

3.2 Mistargeting

Let D^* denote the treatment which was *intended* under the targeted program and D denote treatment *received*. Since the program is mistargeted, D is endogenous such that DID with observed D does not have a causal interpretation and will not recover anything meaningful.[7]

For this setup, $Y_t(D^*)$ denotes the potential outcome at period t corresponding to the *intended* treatment. Then, τ is interpreted as the *average treatment effect for those intended to be treated* $(D^* = 1)$, which we refer to as ATITT. This setting bears similarity to the intention to treat (ITT) analysis. The latter uses the random treatment assignment indicator to estimate the ITT effect since actual treatment adoption is endogenous due to non-compliance. A mistargeting setting is different in that: 1) original treatment assignment is nonrandom, such as an eligibility criteria (as is common with targeted programs like conditional cash transfers etc.) 2) The true assignment is unknown 3) Endogeneity in the observed treatment stems from a misclassification problem.

Given that one interprets τ as the ATITT, the theory developed in section 2 can be applied without further comment.

4 Empirical Applications

In this section, we apply the proposed method to two large-scale national programs in India. The first, known as the Indira Gandhi National Old Age Pension Scheme (IGNOAPS), is a social assistance program whose objective is to provide a monthly pension to elderly persons with no regular income or family support (Kaushal (2014); Unnikrishnan and Imai (2020); Asri (2019)). Benefits from government-run transfer programs are well-known to be misreported (Meyer et al. (2015); Stecklov et al. (2018); Martinelli and Parker (2009)). This is especially true of India, where respondents are well known to under-report asset or income transfers in order to appear more needy

[7] De Chaisemartin and d'Haultfoeuille (2018) study endogenous treatment selection but rather focus on identifying and estimating the local average treatment effect.

(Stecklov et al. (2018)). Based on the 2012 data from Indian Human Development Surveys (IHDS), IGNOAPS's coverage is less than half of that reported in official government statistics. This indicates serious under-reporting of pension benefits.

The second is an in-kind transfer program known as the Targeted Public Distribution System (TPDS). TPDS distributes highly subsidized food grains, mainly rice and wheat, to poor eligible households (Balani (2013); Gadenne et al. (2021)) who are identified based on a selection criteria.[8] Once identified, these households are issued BPL (below poverty line) ration cards which entitles them to receive food grains at much lower prices than non-BPL households. The latter purchase these commodities at market prices.

A key challenge with TPDS is that the program is heavily mistargeted (Dutta and Ramaswami (2001); Swaminathan and Misra (2001); Hirway (2003); Khera (2008)).[9] In particular, errors of *exclusion* are disproportionately larger than errors of *inclusion* (Jha et al. (2013); Pingali et al. (2019)).[10] As a rough estimate, we observe 56% of the poor households in our data having no BPL ration card (wrong exclusion) whereas only 8% of the non-poor households report having a BPL ration card (wrong inclusion) (Appendix Table C.3).

4.1 Data

For both programs, we use data from the IHDS. These are a large scale, multi-topic, nationally representative panel of household surveys conducted by the National Council of Applied Economic Research India, the University of Maryland, Indiana University, and the University of Michigan (Desai and Vanneman (2010, 2018)). IHDS collects in-

[8]The selection criteria includes qualitative variables such as possession of land operated/owned; ownership of TVs, motorcycles, and other durables; and ownership of agricultural machinery and implements (Kochar (2005); Ram et al. (2009); Kaushal and Muchomba (2015)).

[9]Estimates suggest that targeting errors continue to be significant as only 28% of the bottom 40% of the households access TPDS (Pingali et al. (2019)).

[10]For instance, Jha et al. (2013) report that the proportion of poor who used TPDS in 2004-2005 was only 30% and the exclusion error was as high as 70%.

dividual and household level data on various indicators, including household income, expenditure, assets, education, caste, gender relations, health, local infrastructure, and availability of facilities. More importantly, the health module in the IHDS collects anthropometric data on all children up to the age of 11 years present in the household. The IHDS also includes ration card ownership status and records self-reported status on benefits received from various government programs, including old-age pension benefits received under the IGNOAPS.

4.2 Correcting Under-reporting in IGNOAPS

We focus on estimating the impact of a change in IGNOAPS's targeting rule, when the eligibility age was lowered from 65 to 60 years, on the health outcomes of children residing in pension receiving households. Social pension programs have been shown to be important tools for poverty reduction with wide-ranging impacts beyond its direct beneficiaries (Case and Deaton (1998)).[11] For example, Duflo (2003) shows that the South African old-age pension program positively impacted the health outcomes of co-residing children.

In our sample, we observe 660 households that report receiving IGNOAPS pension (treated group) and 6,498 households that do not report receiving it (control group) for years 2005 (baseline) and 2012 (endline). We consider two separate binary outcomes; one that captures whether there are any underweight children in the household aged 2-11 years and the second which captures stunting in children between the same ages. A child is defined as underweight and/or stunted if the weight-for-age z-score (WAZ) and/or height-for-age z-score (HAZ) is less than 2 standard deviations below the World Health Organization's global age-wise weight and height standards. We code the outcome variable as 1 if any children in the household between 2 to 11 years were underweight and/or stunted and 0 otherwise.

We formulate under-reporting of pension benefits under IGNOAPS using the POP

[11] Studies have shown that other members of the households, especially children, benefit from pensions to the elderly (Duflo (2003); Edmonds et al. (2005)).

equation in (9). In this case, D is the self reported IGNOAPS pension benefit status and R contains variables used to determine the eligibility of IGNOAPS beneficiaries, like the number of old people (≥ 60 years) in the household, consumption expenditures, and land and asset ownership. In particular, we use the presence of any non-household members at the time of the interview as the excluded variable (instrument) that predicts S. The argument here is: if a household truly receives a pension, presence of a non-household member during the interview increases the likelihood that the respondent is truthful in their report. Given that households in rural India live in close-knit communities where neighbors and other households in the community know almost everything about each other's background and activities, underreporting benefits from old age pension in the presence of other community members is less likely.[12]

We then implement the proposed two-step DID estimator by plugging in the first-step predicted probability of truly receiving the IGNOAPS pension benefits into the regression specification in (11). The dependent variable is a change in the stunting and underweight status between baseline and endline samples. See Table C.1 for a list of covariates included in this regression.

4.2.1 Results

The results from the first-step estimation of the POP equation are presented in Table 1 and those from the second-stage regression specifications are reported in Table 2. Columns (1)-(4) provide DID estimates using the under-reported IGNOAPS treatment

[12]Individuals may underreport even in the presence of others if there is a social stigma attached to the information that is being elicited. This generally happens in the case of diseases, health problems, and questions of personal and sensitive nature like sexual behavior and domestic violence. Old age pension does not carry any particular social stigma; therefore, we don't believe there is an incentive to underreport pension benefits due to social stigma. We, however, believe that community members will be well aware of the presence of an older member receiving a pension from the government because the program is well known and the average pension amount received by the treated households is INR 370 (around $7) per month, which may be large enough for the households to afford better nutrition for children but small enough for household to try to hide from neighbors and community members.

dummy. We observe that these estimates are negative and statistically insignificant for stunting but positive and statistically significant for being underweight. These uncorrected estimates suggest that while the incidence of stunting declines, incidence of being underweight increases for households receiving pension benefits. Estimates from our proposed procedure are reported in columns (5) and (6) of the same table. We find these to be negative and large (in comparison to uncorrected estimates) for stunting and close to zero for being underweight. However, these are estimated noisily. Our results indicate that the actual effect of receiving pension on children's health is higher than what the naive estimates would suggest.

4.3 Correcting Exclusion Errors in TPDS

For TPDS, we observe 3,035 households having a BPL ration card (treated group) and 6,345 without a BPL ration card (control group). Our outcome of interest is the daily per-capita calorie intake which is calculated from item-wise food consumption data reported in IHDS.[13] We leverage the global food price shock of 2007-08, which caused the market price of rice and wheat to go up, to estimate the ATITT for truly-eligible BPL ration card holders.[14] Our hypothesis is that access to TPDS subsidies will allow BPL cardholders to maintain their real value of income if food prices go up (relative to non-BPL households).

The exclusion errors in targeting of BPL ration cards are modeled using the POP model in (9). In this case, D is the observed BPL ration card status and acts as a mis-targeted proxy for the true but unobserved eligibility status D^*. S is an indicator for correct targeting of an eligible household. R represents a vector of variables that were supposed to be used for determining the true eligibility of TPDS beneficiaries like incomes, consumption expenditures, land and asset ownership. In this particular case, we use links with local officials and membership in local caste associations as the set of

[13] Additional details about sample construction can be found in Appendix B.

[14] Although BPL ration card households have to purchase TPDS grains, they are available at highly subsidized prices which are much lower than the market prices.

excluded variables (instruments) that predict S. The idea here is that conditional on a household being eligible to get a BPL ration card, such links and affiliations increase the likelihood of actually receiving the card.[15]

We then implement the proposed two-step DID estimator by plugging in the predicted probability from the first-step into a FD regression specification where the dependent variable is the change in the daily per-capita calorie intake between baseline and endline. See Table C.2 for a list of covariates included in this regression.

4.3.1 Results

The results from first-step estimation of the POP equation are presented in Table 3 whereas those from the second-stage regression specifications are reported in Table 4. Columns (1)-(3) provide DID estimates that use the mistargeted BPL dummy. While these are positive and statistically significant, these do not have an interesting causal interpretation. Yet, these are the estimates that naive analyses of such a mistargeted program are bound to report. Our main DID estimates are reported in columns (4) and (5) of the same table. According to the theory outlined in this paper, these have a valid causal interpretation and should be interpreted as providing estimates of the effect of being truly eligible to receive BPL subsidy on the average calorie intake of poor eligible households. We find this effect to be negative and significant. We also find our estimates robust to including controls and benefits received with other governmental programs.

5 Conclusion

Much of the DID literature, both econometric and applied, assumes that the treatment is measured accurately. There is sufficient evidence in the applied literature to suggest

[15] This is consistent with observations made in the literature that elite capture and connections with local politicians is helpful in getting access to benefits from transfer programs in developing countries (Pande (2007); Besley et al. (2012); Panda (2015); Gambhir et al. (2017)). For TPDS, Panda (2015) shows that local political connections are conducive to being selected into getting a BPL ration card.

that participation in programs is mismeasured. This may be due to misreporting of program benefits or mistargeting of interventions which results in the wrongful inclusion or exclusion of program beneficiaries. In this paper, we focus on identifying and estimating the latent ATT when the observed treatment is misclassified for the true (but latent) treatment variable in a standard DID framework.

Our solution considers the common case of one-sided misclassification where we allow such errors to be differential. Identification of the latent ATT is achieved by characterizing these errors using a partial observability probit that involves an exclusion restriction which only affects the misclassification probability. The proposed procedure, then, corrects for the problem by predicting the latent treatment from the first-step and plugging it into DID regressions in the second-step. The resulting two-step estimator is shown to have favorable asymptotic properties.

We demonstrate the method by applying it to two large-scale transfer programs in India. The first one is an old age pension scheme whose benefits are known to be under-reported and the second is a targeted food assistance program which is well-known to be mistargeted (excludes the eligible poor). After correcting for under-reporting of benefits, we find that households receiving pensions had fewer stunted children. This effect is more muted when we do not address the under-reporting of pension benefits. In the second application, the naive (uncorrected) estimates suggest that receiving the food subsidy improved the nutritional outcomes of BPL ration card households. Due to under targeting of BPL ration card beneficiaries, this estimate does not have a causal interpretation. Therefore, we focus on estimates of the average intended effect of such a subsidy on eligible households' nutritional outcomes. We find this effect to be negative and statistically significant. Overall, these two applications help to highlight how using a mismeasured treatment in a DID context may be misguided for policy making purposes. We argue that the method proposed in this paper can be applied to situations where the treatment may either be misreported or mistargeted and how, depending on the application, one may correctly causally interpret the two-step estimates from our procedure.

References

Abadie, A. (2005). Semiparametric difference-in-differences estimators. *The Review of Economic Studies*, 72(1):1–19.

Acerenza, S., Ban, K., and Kédagni, D. (2021). Marginal treatment effects with misclassified treatment. *arXiv preprint arXiv:2105.00358*.

Aigner, D. J. (1973). Regression with a binary independent variable subject to errors of observation. *Journal of Econometrics*, 1(1):49–59.

Alam, M. (2004). Ageing, old age income security and reforms: An exploration of indian situation. *Economic and Political Weekly*, pages 3731–3740.

Alatas, V., Banerjee, A., Hanna, R., Olken, B. A., and Tobias, J. (2012). Targeting the poor: evidence from a field experiment in indonesia. *American Economic Review*, 102(4):1206–40.

Asri, V. (2019). Targeting of social transfers: are india's poor older people left behind? *World Development*, 115:46–63.

Balani, S. (2013). Functioning of the public distribution system. *PRS Legislative Research, New Delhi*.

Battistin, E. and Sianesi, B. (2011). Misclassified treatment status and treatment effects: an application to returns to education in the united kingdom. *Review of Economics and Statistics*, 93(2):495–509.

Besley, T., Pande, R., and Rao, V. (2012). Just rewards? local politics and public resource allocation in south india. *The World Bank Economic Review*, 26(2):191–216.

Black, D. A., Berger, M. C., and Scott, F. A. (2000). Bounding parameter estimates with nonclassical measurement error. *Journal of the American Statistical Association*, 95(451):739–748.

Bollinger, C. R. (1996). Bounding mean regressions when a binary regressor is mismeasured. *Journal of Econometrics*, 73(2):387–399.

Botosaru, I. and Gutierrez, F. H. (2018). Difference-in-differences when the treatment status is observed in only one period. *Journal of Applied Econometrics*, 33(1):73–90.

Bound, J., Brown, C., and Mathiowetz, N. (2001). Measurement error in survey data. In *Handbook of Econometrics*, volume 5, pages 3705–3843. Elsevier.

Bruckmeier, K., Riphahn, R. T., and Wiemers, J. (2021). Misreporting of program take-up in survey data and its consequences for measuring non-take-up: new evidence from linked administrative and survey data. *Empirical Economics*, 61(3):1567–1616.

Buchmueller, T. C., DiNardo, J., and Valletta, R. G. (2011). The effect of an employer health insurance mandate on health insurance coverage and the demand for labor: Evidence from hawaii. *American Economic Journal: Economic Policy*, 3(4):25–51.

Callaway, B. and Sant'Anna, P. H. (2020). Difference-in-differences with multiple time periods. *Journal of Econometrics*.

Calvi, R., Lewbel, A., and Tommasi, D. (2021). Late with missing or mismeasured treatment. *Journal of Business & Economic Statistics*, 0(ja):1–45.

Cameron, L. and Shah, M. (2014). Can mistargeting destroy social capital and stimulate crime? evidence from a cash transfer program in indonesia. *Economic Development and Cultural Change*, 62(2):381–415.

Case, A. and Deaton, A. (1998). Large cash transfers to the elderly in south africa. *The economic journal*, 108(450):1330–1361.

Celhay, P., Meyer, B. D., and Mittag, N. (2024). What leads to measurement errors? evidence from reports of program participation in three surveys. *Journal of Econometrics*, 238(2):105581.

Cornia, G. A. and Stewart, F. (1993). Two errors of targeting. *Journal of International Development*, 5(5):459–496.

De Chaisemartin, C. and d'Haultfoeuille, X. (2018). Fuzzy differences-in-differences. *The Review of Economic Studies*, 85(2):999–1028.

Denteh, A. and Kédagni, D. (2022). Misclassification in difference-in-differences models. *arXiv preprint arXiv:2207.11890*.

Desai, S. and Vanneman, R. (2010). National Council of Applied Economic Research, New Delhi. India Human Development Survey (IHDS), 2005. *Ann Arbor, MI: Inter-university Consortium for Political and Social Research*.

Desai, S. and Vanneman, R. (2018). National Council of Applied Economic Research, New Delhi. India Human Development Survey (IHDS), 2012. *Ann Arbor, MI: Inter-university Consortium for Political and Social Research, University of Michigan*.

DiTraglia, F. J. and Garcia-Jimeno, C. (2019). Identifying the effect of a mis-classified, binary, endogenous regressor. *Journal of Econometrics*, 209(2):376–390.

Duflo, E. (2003). Grandmothers and granddaughters: old-age pensions and intrahousehold allocation in south africa. *The World Bank Economic Review*, 17(1):1–25.

Dutta, B. and Ramaswami, B. (2001). Targeting and efficiency in the public distribution system: Case of andhra pradesh and maharashtra. *Economic and Political Weekly*, pages 1524–1532.

Edmonds, E. V., Mammen, K., and Miller, D. L. (2005). Rearranging the family?: Income support and elderly living arrangements in a low-income country. *Journal of Human resources*, 40(1):186–207.

Frazis, H. and Loewenstein, M. A. (2003). Estimating linear regressions with mismeasured, possibly endogenous, binary explanatory variables. *Journal of Econometrics*, 117(1):151–178.

Gadenne, L., Norris, S., Singhal, M., and Sukhtankar, S. (2021). In-kind transfers as insurance. Technical report, National Bureau of Economic Research.

Gambhir, I., Desai, S., and Vanneman, R. (2017). Role of networks in the elite capture of food subsidy program in india. In *International Union for the Scientific Study of Population (IUSSP) XXVIII International Population Conference was held*, volume 29.

Groen, J. A. and Polivka, A. E. (2008). The effect of hurricane katrina on the labor market outcomes of evacuees. *American Economic Review*, 98(2):43–48.

Haider, S. J. and Stephens Jr, M. (2020). Correcting for misclassified binary regressors using instrumental variables. Technical report, National Bureau of Economic Research.

Hirway, I. (2003). Identification of bpl households for poverty alleviation programmes. *Economic and Political Weekly*, pages 4803–4808.

Jha, S., Kotwal, A., and Ramaswami, B. (2013). Safety nets and food programs in asia: A comparative perspective. *Asian Development Bank Economics Working Paper Series*, (371).

Kane, T. J., Rouse, C. E., and Staiger, D. O. (1999). Estimating returns to schooling when schooling is misreported.

Kaushal, N. (2014). How public pension affects elderly labor supply and well-being: Evidence from india. *World Development*, 56:214–225.

Kaushal, N. and Muchomba, F. M. (2015). How consumer price subsidies affect nutrition. *World Development*, 74:25–42.

Khera, R. (2008). Access to the targeted public distribution system: a case study in rajasthan. *Economic and Political Weekly*, pages 51–56.

Kochar, A. (2005). Can targeted food programs improve nutrition? an empirical analysis of india's public distribution system. *Economic development and cultural change*, 54(1):203–235.

Kreider, B. (2010). Regression coefficient identification decay in the presence of infrequent classification errors. *The Review of Economics and Statistics*, 92(4):1017–1023.

Lewbel, A. (2007). Estimation of average treatment effects with misclassification. *Econometrica*, 75(2):537–551.

Loewenstein, M. A. and Spletzer, J. R. (1997). Delayed formal on-the-job training. *ILR Review*, 51(1):82–99.

Mahajan, A. (2006). Identification and estimation of regression models with misclassification. *Econometrica*, 74(3):631–665.

Martinelli, C. and Parker, S. W. (2009). Deception and misreporting in a social program. *Journal of the European Economic Association*, 7(4):886–908.

Meyer, B. D. and Mittag, N. (2017). Misclassification in binary choice models. *Journal of Econometrics*, 200(2):295–311.

Meyer, B. D., Mok, W. K., and Sullivan, J. X. (2015). Household surveys in crisis. *Journal of Economic Perspectives*, 29(4):199–226.

Millimet, D. L. (2011). The elephant in the corner: a cautionary tale about measurement error in treatment effects models. In *Missing data methods: Cross-sectional methods and applications*. Emerald Group Publishing Limited.

Nguimkeu, P., Denteh, A., and Tchernis, R. (2019). On the estimation of treatment effects with endogenous misreporting. *Journal of Econometrics*, 208(2):487–506.

Niehaus, P. and Sukhtankar, S. (2013). Corruption dynamics: The golden goose effect. *American Economic Journal: Economic Policy*, 5(4):230–269.

Panda, S. (2015). Political connections and elite capture in a poverty alleviation programme in india. *The Journal of Development Studies*, 51(1):50–65.

Pande, R. (2007). Understanding political corruption in low income countries. *Handbook of development economics*, 4:3155–3184.

Pingali, P., Aiyar, A., Abraham, M., and Rahman, A. (2019). Reimagining safety net programs. In *Transforming food systems for a rising India*, pages 135–164. Springer.

Poirier, D. J. (1980). Partial observability in bivariate probit models. *Journal of Econometrics*, 12(2):209–217.

Ram, F., Mohanty, S., and Ram, U. (2009). Understanding the distribution of bpl cards: all-india and selected states. *Economic and Political Weekly*, pages 66–71.

Sant'Anna, P. H. and Zhao, J. (2020). Doubly robust difference-in-differences estimators. *Journal of Econometrics*, 219(1):101–122.

Sjoquist, D. L. and Winters, J. V. (2015). State merit aid programs and college major: A focus on stem. *Journal of Labor Economics*, 33(4):973–1006.

Stecklov, G., Weinreb, A., and Carletto, C. (2018). Can incentives improve survey data quality in developing countries?: results from a field experiment in india. *Journal of the Royal Statistical Society Series A: Statistics in Society*, 181(4):1033–1056.

Swaminathan, M. and Misra, N. (2001). Errors of targeting: Public distribution of food in a maharashtra village, 1995-2000. *Economic and Political Weekly*, pages 2447–2454.

Tallis, G. M. (1961). The moment generating function of the truncated multi-normal distribution. *Journal of the Royal Statistical Society: Series B (Methodological)*, 23(1):223–229.

Tommasi, D. and Zhang, L. (2024). Bounding program benefits when participation is misreported. *Journal of Econometrics*, 238(1):105556.

Unnikrishnan, V. and Imai, K. S. (2020). Does the old-age pension scheme improve household welfare? evidence from india. *World Development*, 134:105017.

Ura, T. (2018). Heterogeneous treatment effects with mismeasured endogenous treatment. *Quantitative Economics*, 9(3):1335–1370.

Wooldridge, J. M. (2021). Two-way fixed effects, the two-way mundlak regression, and difference-in-differences estimators. *Available at SSRN 3906345*.

Yanagi, T. (2019). Inference on local average treatment effects for misclassified treatment. *Econometric Reviews*, 38(8):938–960.

Tables

Table 1: Partial Observability Model for Self-Reported IGNOAPS Benefits Status

	(1)	(2)
Log operated land	-0.058	0.368*
	(0.112)	(0.207)
Log consumption expenditure	0.045	-0.187
	(0.107)	(0.124)
Asset count	-0.042***	0.044*
	(0.010)	(0.024)
BPL ration card dummy	0.293***	0.141
	(0.074)	(0.119)
Muslim dummy	-0.062	
	(0.147)	
Other religion summy	0.769***	
	(0.278)	
SC/ST dummy	0.304**	0.069
	(0.152)	(0.150)
Other backward caste dummy	0.248**	0.246*
	(0.099)	(0.127)
Number of old members	0.214***	1.526**
	(0.074)	(0.701)
Non-household member present during survey dummy		0.399***
		(0.142)
N	7158	

Notes: Columns (1) and (2) report estimates from the POP equation for D^* and S equations, respectively. The implementation of the first step is done using the `biprobit` command in Stata. The dependent variable in both equations is a dummy variable which is 1 if the household is treated (reported geting pension under the IGNOAPS), 0 otherwise. Standard errors in parentheses clustered at the village level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 2: Difference-in-Difference Estimates for IGNOAPS

	(1)	(2)	(3)	(4)	(5)	(6)
	Observed treatment				Predicted treatment	
Dependent variable:	Stunted	Underweight	Stunted	Underweight	Stunted	Underweight
IGNOAPS	-0.001	0.080***	-0.025	0.051*	-0.059	0.002
	(0.026)	(0.027)	(0.026)	(0.027)	(0.106)	(0.087)
Controls	No	No	Yes	Yes	Yes	Yes
Controls $\times$ IGNOAPS	No	No	Yes	Yes	Yes	Yes
N	7158	7158	7158	7158	7158	7158

Notes: IGNOAPS is a dummy variable which is 1 if the respondent reported a member recieving pension under the IGNOAPS, 0 otherwise. Household controls include log operated land, log consumption expenditure, number of assets, caste dummies, religion dummies, literate member dummy, dummy for participation in the Mahatma Gandhi National Rural Guarantee Scheme (MGNREGS), number of children, number of elders and average age of children who were surveyed for anthropometric measures. All covariates demeaned by treatment group means. Standard errors in parentheses for specifications (1) to (4) are clustered at the village level. Specifications (5) and (6) have bootstrapped standard errors with 200 replications. ***, **,and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 3: Partial Observability Model for BPL Ration Card Ownership

	(1)	(2)
Log operated land	-0.074	-0.570***
	(0.093)	(0.117)
Log consumption expenditure	-0.386***	-0.041
	(0.112)	(0.154)
Hindu dummy	0.829***	
	(0.175)	
Other caste dummy	-0.988***	
	(0.154)	
Other backward caste dummy	-0.306**	
	(0.120)	
Rural dummy	0.347	0.325
	(0.211)	(0.265)
Any vehicle owned dummy	0.004	-0.184
	(0.185)	(0.153)
Motor vehicle owned dummy	-1.413***	6.817***
	(0.276)	(2.043)
Cooler owned dummy	-0.300	-0.406
	(0.206)	(0.276)
TV owned dummy	-0.458***	0.102
	(0.152)	(0.172)
Electric fan owned dummy	-0.264	-0.263**
	(0.162)	(0.132)
Refrigerator owned dummy	-0.142	-0.886***
	(0.205)	(0.307)
Kaccha house dummy	-0.253**	-0.256**
	(0.120)	(0.116)
LPG gas dummy	0.785***	-0.172
	(0.257)	(0.322)
Tractor/thresher owned dummy	-0.551*	-0.662*
	(0.289)	(0.339)
Any member in local caste associations		0.680***
		(0.117)
Panchayat official close to household		0.276***
		(0.106)
N	9380	

Notes: Columns (1) and (2) report estimates from the POP equation for D^* and S equations, respectively. The implementation of the first-step is done using the `biprobit` command in Stata. The dependent variable in both equations is a dummy variable which is 1 if the household is treated (owns a below poverty line ration card), 0 otherwise. Standard errors in parentheses clustered at the village level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4: Difference-in-Difference Estimates for TPDS

	(1)	(2)	(3)	(4)	(5)
	Observed treatment			Predicted treatment	
Dependent variable: Log change in calories consumed per person per day					
BPL	0.042***	0.034***	0.027**	-0.070*	-0.068*
	(0.016)	(0.013)	(0.013)	(0.038)	(0.038)
Controls	No	Yes	Yes	Yes	Yes
Government programs	No	Yes	Yes	Yes	Yes
Controls $\times$ BPL	No	No	Yes	No	Yes
Government programs $\times$ BPL	No	No	Yes	No	Yes
N	9380	9380	9380	9380	9380

Notes: BPL is a dummy variable which is 1 if the household owns a below poverty line ration card, 0 otherwise. Household controls include log operated land, log consumption expenditure, caste dummies, number of family members, rural dummy, asset count, dummies for ownership of vehicle, motor car, moterbike, cooler or AC, refrigerator, TV, electric fan, telephone, electricity and finally a dummy for whether the household have a makeshift dwelling or a *kaccha* house. Government programs include participation in the Mahatma Gandhi National Rural Guarantee Scheme (MGNREGS) or benefits received from other welfare programs like health insurance, scholarships, old age pension, maternity scheme, disability scheme, income generation programs other than MGNREGS, assistance from drought/flood compensation, and insurance payouts. All covariates demeaned by treatment group means. Standard errors in parentheses for specifications (1) and (3) are clustered at the village level. Specifications (4) and (5) have bootstrapped standard errors with 200 replications. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Online Supplementary Material

Akanksha Negi[†] and Digvijay S. Negi[‡]

A Proofs

Proof of Proposition 1. For simplicity, assume there are no covariates and that the no-anticipation and parallel trends assumptions hold unconditionally. Using equation 1, we can write,

$$\Delta Y = \Delta Y(0) + (Y_1(1) - Y_1(0)) \cdot D^* - (Y_0(1) - Y_0(0)) \cdot D^*$$

Parallel trends implies $\mathbb{E}[\Delta Y(0)|D^*] = \mathbb{E}[\Delta Y(0)] = \theta$ (say). Then, by parallel trends and no-anticipation assumption

$$\mathbb{E}[\Delta Y|D^*] = \theta + D^* \cdot \tau. \tag{A.1}$$

Alternatively, one may write (A.1) in error form as $\Delta Y = \theta + D^* \cdot \tau + \Delta\xi$. Using this, the unconditional DID becomes

$$\mathbb{E}[\Delta Y|D=1] - \mathbb{E}[\Delta Y|D=0] = \tau \cdot q_0 + \mathbb{E}[\Delta\xi|D=1] - \mathbb{E}[\Delta\xi|D=0] \tag{A.2}$$

where the above equality uses $q_1 = 1$. If misreporting is non-differential i.e. $\mathbb{E}[\Delta\xi|D, D^*] = \mathbb{E}[\Delta\xi|D^*]$, then

$$\mathbb{E}[\Delta\xi|D=1] - \mathbb{E}[\Delta\xi|D=0] = \Big(\mathbb{E}(\Delta\xi|D^*=1) - \mathbb{E}(\Delta\xi|D^*=0)\Big) \cdot q_0 = 0$$

Therefore, $\mathbb{E}[\Delta Y|D=1] - \mathbb{E}[\Delta Y|D=0] = \tau \cdot q_0$. $\square$

Proof of Theorem 1.

Consistency of $\hat{\theta}^{2S}_{FD}$: Consider

$$\begin{aligned}
\hat{\theta}^{2S}_{FD} - \theta &= \left(\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N}\right)^{-1} \frac{\hat{W}^{*\prime}\hat{M}^*\Delta\varepsilon}{N} \\
&= \left(\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N}\right)^{-1} \frac{\hat{W}^{*\prime}\hat{M}^*}{N}\left[(\mathring{R} - \hat{R}^*)\delta + (W^* - \hat{W}^*)\theta + \Delta\xi\right] \\
&= \left(\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N}\right)^{-1} \left[\frac{\hat{W}^{*\prime}\hat{M}^*\mathring{R}\delta}{N} + \frac{\hat{W}^{*\prime}\hat{M}^*(W^* - \hat{W}^*)\theta}{N} + \frac{\hat{W}^{*\prime}\hat{M}^*\Delta\xi}{N}\right]
\end{aligned} \tag{A.3}$$

Now $\hat{R}^* \xrightarrow{p} \mathring{R}$ because

$$\hat{\bar{X}}_1^* = \frac{1}{N} \cdot \frac{N}{\hat{N}^*} \sum_{i=1}^{N} \hat{D}_i^* \cdot X_i = \frac{1}{\mathbb{P}(D_i^* = 1)} \cdot \frac{1}{N} \sum_{i=1}^{N} \hat{D}_i^* \cdot X_i + o_p(1)$$
$$= \frac{1}{\mathbb{P}(D_i^* = 1)} \cdot \mathbb{E}(D_i^* X_i) + o_p(1)$$
$$= \mathbb{E}(X_i | D_i^* = 1) + o_p(1)$$

where the second equality follows from the fact that $\hat{D}^* = \Phi(R\hat{\gamma}) \xrightarrow{p} \Phi(R\gamma)$. This implies that as $N \to \infty$,

$$\hat{\bar{X}}_1^* \xrightarrow{p} \mathbb{E}(X_i | D_i^* = 1)$$

Hence,

$$\left(\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N}\right)^{-1} \xrightarrow{p} \mathbb{E}[\Phi^2(R_i\gamma)\mathring{R}_i'\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i]$$

and

$$\begin{aligned} \frac{\hat{W}^{*\prime}\hat{M}^*\mathring{R}\delta}{N} &\xrightarrow{p} \left\{\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\mathbb{E}[\mathring{R}_i'\mathring{R}_i]\right\}\delta \\ &= 0 \end{aligned} \tag{A.4}$$

Now,

$$\begin{aligned} \frac{\hat{W}^{*\prime}\hat{M}^*(W^* - \hat{W}^*)\theta}{N} &\xrightarrow{p} \Big\{\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'(D_i^* - \Phi(R_i\gamma))\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1} \\ &\quad \mathbb{E}[\mathring{R}_i'(D_i^* - \Phi(R_i\gamma))\mathring{R}_i]\Big\}\theta \\ &= 0 \end{aligned} \tag{A.5}$$

where the last equality is due to law of iterated expectations because $\mathbb{E}[D_i^* - \Phi(R_i\gamma)|R_i] = 0$. Finally,

$$\begin{aligned} \frac{\hat{W}^{*\prime}\hat{M}^*\Delta\xi}{N} &\xrightarrow{p} \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\Delta\xi_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\mathbb{E}[\mathring{R}_i'\Delta\xi_i] \\ &= 0 \end{aligned}$$

where again the last equality follows due to $\mathbb{E}(\mathring{R}_i'\Delta\xi_i) = 0$ and law of iterated expectations.

Therefore, $\hat{\theta}_{\text{FD}}^{2\text{S}} - \theta \xrightarrow{p} 0$. Now,

$$\hat{\tau}_{\text{FD}}^{2\text{S}} = \frac{1}{\hat{N}^*}\sum_{i=1}^{N}\hat{D}_i^*\hat{R}_i^*\hat{\theta}_{\text{FD}}^{2\text{S}} = \frac{N}{\hat{N}^*}\left\{\frac{1}{N}\sum_{i=1}^{N}\hat{D}_i^*\hat{R}_i^*\theta + \frac{1}{N}\sum_{i=1}^{N}\hat{D}_i^*\hat{R}_i^*\left(\hat{\theta}_{\text{FD}}^{2\text{S}} - \theta\right)\right\}$$

Now since $\frac{\hat{N}^*}{N} = \mathbb{P}(D_i^* = 1) + o_p(1)$ and $\frac{1}{N}\sum_{i=1}^{N}\hat{D}_i^*\hat{R}_i^* = \frac{1}{N}\sum_{i=1}^{N}\Phi(R_i\gamma)\mathring{R}_i + o_p(1) =$

$\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i] + o_p(1)$. Therefore,

$$\hat{\tau}^{2S}_{\text{FD}} = \frac{1}{\mathbb{P}(D^*_i = 1)} \left\{ \frac{1}{N}\sum_{i=1}^{N} \Phi(R_i\gamma)\mathring{R}_i\theta + \frac{1}{N}\sum_{i=1}^{N} \Phi(R_i\gamma)\mathring{R}_i \left(\hat{\theta}^{2S}_{\text{FD}} - \theta\right) \right\} + o_p(1)$$

Now, $\frac{1}{N}\sum_{i=1}^{N} \Phi(R_i\gamma)\mathring{R}_i\theta = \mathbb{E}[\mathbb{P}(D^*_i = 1|R_i)\mathring{R}_i] + o_p(1)$ where $\mathbb{E}[\mathbb{P}(D^*_i = 1|R_i)\mathring{R}_i] = \mathbb{E}[\mathbb{E}(D^*_i\mathring{R}_i|R_i)] = \mathbb{E}[D^*_i\mathring{R}_i]$. Hence,

$$\text{plim}(\hat{\tau}^{2S}_{\text{FD}} - \tau) = \mathbb{E}[\mathring{R}_i \cdot \text{plim}(\hat{\theta}^{2S}_{\text{FD}} - \theta)|D^*_i = 1] = 0$$

Therefore, $\hat{\tau}^{2S}_{\text{FD}} - \tau \xrightarrow{p} 0$.

Consistency of $\hat{\theta}^{2S}_{\text{POLS}}$: Let $\hat{\pi}^{2S}_2$ be the two-step estimator of the coefficient on $\hat{W}^*_i$ from estimating the regression of

$$Y_i \text{ on } \hat{R}^*_i, \hat{W}^*_i \text{ if } T_i = 1 \text{ for } i = 1, \dots, N$$

which again using Frisch-Waugh gives us the following estimating equation

$$\hat{M}^*_1 Y_1 = \hat{M}^*_1\hat{W}^*_1 + \hat{M}^*_1\varepsilon_1 \tag{A.6}$$

where $\hat{M}^*_1 = I_1 - \hat{R}^*_1(\hat{R}^{*\prime}_1\hat{R}^*_1)^{-1}\hat{R}^{*\prime}_1$ is the residual making matrix for $\hat{R}^*_1$ whose i-th element is given as $T_i\hat{R}^*_i$ and $\hat{W}^*_1$, ε_1 are defined analogously.

Then,

$$\begin{aligned}
\hat{\pi}^{2S}_2 - \pi_2 &= \left(\frac{\hat{W}^{*\prime}_1\hat{M}^*_1\hat{W}^*_1}{N}\right)^{-1} \frac{\hat{W}^{*\prime}_1\hat{M}^*_1\varepsilon_1}{N} \\
&= \left(\frac{\hat{W}^{*\prime}_1\hat{M}^*_1\hat{W}^*_1}{N}\right)^{-1} \frac{\hat{W}^{*\prime}_1\hat{M}^*_1}{N}\left\{(\mathring{R}_1 - \hat{R}^*_1)\pi_1 + (W^*_1 - \hat{W}^*_1)\pi_2 + \xi^1\right\} \\
&= \left(\frac{\hat{W}^{*\prime}_1\hat{M}^*_1\hat{W}^*_1}{N}\right)^{-1} \left\{\frac{\hat{W}^{*\prime}_1\hat{M}^*_1\mathring{R}_1\pi_1}{N} + \frac{\hat{W}^{*\prime}_1\hat{M}^*_1(W^*_1 - \hat{W}^*_1)\pi_2}{N} + \frac{\hat{W}^{*\prime}_1\hat{M}^*_1\xi^1}{N}\right\}
\end{aligned} \tag{A.7}$$

where

$$\left(\frac{\hat{W}^{*\prime}_1\hat{M}^*_1\hat{W}^*_1}{N}\right)^{-1} \xrightarrow{p} \lambda\left\{\mathbb{E}[\Phi^2(R_i\gamma)\mathring{R}^{\prime}_i\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}^{\prime}_i\mathring{R}_i][\mathbb{E}(\mathring{R}^{\prime}_i\mathring{R}_i)]^{-1}\mathbb{E}[\Phi(R_i\gamma)\mathring{R}^{\prime}_i\mathring{R}_i]\right\} \tag{A.8}$$

and

$$
\begin{aligned}
\frac{\hat{W}_1^{*\prime}\hat{M}_1^*\mathring{R}_1\pi_1}{N} &= \frac{\hat{W}_1^{*\prime}\{I_1-\hat{R}_1^*(\hat{R}_1^{*\prime}\hat{R}_1^*)^{-1}\hat{R}_1^{*\prime}\}\mathring{R}_1\pi_1}{N} \\
&= \left\{\frac{\hat{W}_1^{*\prime}I_1\mathring{R}_1}{N} - \frac{\hat{W}_1^{*\prime}\hat{R}_1^*(\hat{R}_1^{*\prime}\hat{R}_1^*)^{-1}\hat{R}_1^{*\prime}\mathring{R}_1}{N}\right\}\pi_1 \\
&= \left\{\frac{1}{N}\sum_{i=1}^N T_i\hat{W}_i^{*\prime}\mathring{R}_i - \left(\frac{1}{N}\sum_{i=1}^N T_i\hat{W}_i^{*\prime}\hat{R}_i^*\right)\left(\frac{1}{N}\sum_{i=1}^N T_i\hat{R}_i^{*\prime}\hat{R}_i^*\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^N T_i\hat{R}_i^{*\prime}\mathring{R}_i\right)\right\}\pi_1 \\
&\xrightarrow{p} \lambda\left\{\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\mathbb{E}[\mathring{R}_i'\mathring{R}_i]\right\}\pi_1 \\
&= 0
\end{aligned} \tag{A.9}
$$

Similarly,

$$
\begin{aligned}
\frac{\hat{W}_1^{*\prime}\hat{M}_1^*(W_1^*-\hat{W}_1^*)\pi_2}{N} &\xrightarrow{p} \lambda\Big\{\mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'(D_i^*-\Phi(R_i\gamma))\mathring{R}_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\cdot \\
&\qquad \mathbb{E}[\mathring{R}_i'(D_i^*-\Phi(R_i\gamma))\mathring{R}_i]\Big\}\pi_2 \\
&= 0
\end{aligned} \tag{A.10}
$$

where the equality follows from the law of iterated expectations and the fact that $D_i^* \xrightarrow{p} \Phi(R_i\gamma)$. Finally,

$$
\begin{aligned}
\frac{\hat{W}_1^{*\prime}\hat{M}_1^*\xi^1}{N} &\xrightarrow{p} \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\xi_i] - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i][\mathbb{E}(\mathring{R}_i'\mathring{R}_i)]^{-1}\mathbb{E}[\mathring{R}_i'\xi_i] \\
&= 0
\end{aligned} \tag{A.11}
$$

where again the equality follows due to $\mathbb{E}(\mathring{R}_i'\xi_i) = 0$ and law of iterated expectations. Therefore, together with A.7, A.8, A.9, A.10, and A.11, we obtain $\hat{\pi}_2^{2S} - \pi_2 \xrightarrow{p} 0$. Following in a similar manner, let $\hat{\eta}_2^{2S}$ be the two-step estimator of the coefficient on $\hat{W}_i^*$ from estimating the regression of

$$Y_i \text{ on } \hat{R}_i^*, \hat{W}_i^* \text{ if } T_i = 0 \text{ for } i = 1, \ldots, N$$

Then, we can also show that $\hat{\eta}_2^{2S} - \eta_2 \xrightarrow{p} 0$. Now since $\hat{\theta}_{\text{POLS}}^{2S} = \hat{\pi}_2^{2S} - \hat{\eta}_2^{2S}$, therefore $\hat{\theta}_{\text{POLS}}^{2S} - \theta \xrightarrow{p} 0$. Similar to the case for the FD-estimator, we can show $\hat{\tau}_{\text{POLS}}^{2S} - \tau \xrightarrow{p} 0$.

Asymptotic normality for $\hat{\theta}_{\text{FD}}^{2S}$:

$$\sqrt{N}(\hat{\theta}_{\text{FD}}^{2S} - \theta) = \left(\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N}\right)^{-1}\frac{\hat{W}^{*\prime}\hat{M}^*}{\sqrt{N}}\left[(\mathring{R}-\hat{R}^*)\delta + (W^*-\hat{W}^*)\theta + \Delta\xi\right]$$

Now we have already shown that

$$\frac{\hat{W}^{*\prime}\hat{M}^*\hat{W}^*}{N} = \frac{1}{N}\sum_{i=1}^{N}\Phi^2(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^* - \left(\frac{1}{N}\sum_{i=1}^{N}\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*\right)\left(\frac{1}{N}\sum_{i=1}^{N}\hat{R}_i^{*\prime}\hat{R}_i^*\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*\right)$$
$$= \frac{1}{N}\sum_{i=1}^{N}\hat{\Gamma}_i\hat{\Gamma}_i' \xrightarrow{p} \mathbb{E}[\Gamma_i\Gamma_i'] \equiv \Omega_\Gamma$$

where $\hat{\Gamma}_i = \Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime} - \left(\frac{\sum_{i=1}^N \Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*}{N}\right)\left(\frac{\sum_{i=1}^N \hat{R}_i^{*\prime}\hat{R}_i^*}{N}\right)^{-1}\hat{R}_i^{*\prime}$ and $\Gamma_i = \Phi(R_i\gamma)\mathring{R}_i' - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i]\mathbb{E}[\mathring{R}_i'\mathring{R}_i]^{-1}\mathring{R}_i'$. Therefore, using the above we can write

$$\sqrt{N}(\hat{\theta}^{2S}_{\text{FD}} - \theta) = \Omega_\Gamma^{-1}\left(\frac{\hat{W}^{*\prime}\hat{M}^*}{\sqrt{N}}\left[(\mathring{R} - \hat{R}^*)\delta + (W^* - \hat{W}^*)\theta + \Delta\xi\right]\right) + o_p(1) \tag{A.12}$$

Define, $\Pi_i^*(\gamma) = \Phi(R_i\gamma)\mathring{R}_i$ and let $\Pi^*(\gamma) = [\Pi_1^*(\gamma)', \ldots, \Pi_N^*(\gamma)']'$. Then, we can express the terms inside the bracket as

$$\begin{aligned}
&\frac{\hat{W}^{*\prime}\hat{M}^*}{\sqrt{N}}\left[(\mathring{R} - \hat{R}^*)\delta + (W^* - \hat{W}^* - \Pi^*(\gamma) + \Pi^*(\gamma))\theta + \Delta\xi\right] \\
&= \frac{\hat{W}^{*\prime}\hat{M}^*}{\sqrt{N}}\left[(\mathring{R} - \hat{R}^*)\delta + (W^* - \Pi^*(\gamma))\theta + (\Pi^*(\gamma) - \hat{W}^*)\theta + \Delta\xi\right] \\
&= \frac{\hat{W}^{*\prime}\hat{M}^*\mathring{R}\delta}{\sqrt{N}} + \frac{\hat{W}^{*\prime}\hat{M}^*\{(W^* - \Pi^*(\gamma))\theta + \Delta\xi\}}{\sqrt{N}} + \frac{\hat{W}^{*\prime}\hat{M}^*(\Pi^*(\gamma) - \hat{W}^*)\theta}{\sqrt{N}} \\
&= \sqrt{N}H_{1N} + \sqrt{N}H_{2N} + \sqrt{N}H_{3N}
\end{aligned} \tag{A.13}$$

Then by the central limit theorem,

$$\sqrt{N}H_{1N} = \frac{\hat{W}^{*\prime}\hat{M}^*\mathring{R}\delta}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\hat{\Gamma}_i\mathring{R}_i\delta = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\mathring{R}_i\delta + o_p(1) \qquad (\text{where } \mathbb{E}[\Gamma_i\mathring{R}_i] = 0)$$

$$\sqrt{N}H_{2N} = \frac{\hat{W}^{*\prime}\hat{M}^*[(W^* - \Pi^*(\gamma))\theta + \Delta\xi]}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\{(W_i^* - \Pi_i^*(\gamma))\theta + \Delta\xi_i\} + o_p(1)$$

due to law of iterated expectations and the fact that $\mathbb{E}[\Delta\xi|D^*, X] = 0$ and

$$\sqrt{N}H_{3N} = \frac{\hat{W}^{*\prime}\hat{M}^*(\Pi^*(\gamma) - \hat{W}^*)\theta}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\hat{\Gamma}_i\{\Pi_i^*(\gamma) - \hat{W}_i^*\}\theta = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\{\Pi_i^*(\gamma) - \hat{W}_i^*\}\theta + o_p(1)$$

$$\begin{aligned}
&= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\{\Phi(R_i\gamma)\mathring{R}_i - \hat{D}_i^*(\hat{R}_i^* - \mathring{R}_i + \mathring{R}_i)\}\theta + o_p(1) \\
&= \frac{1}{N}\sum_{i=1}^{N}\Gamma_i \cdot \{\sqrt{N}(\Phi(R_i\gamma) - \hat{D}_i^*)\mathring{R}_i - \Phi(R_i\gamma)\cdot\sqrt{N}(\hat{R}_i^* - \mathring{R}_i)\}\theta + o_p(1)
\end{aligned}$$

By the delta method, $\sqrt{N}(\Phi(R_i\gamma) - \hat{D}_i^*) \approx -\phi(R_i\tilde{\gamma})R_i \cdot \sqrt{N}(\hat{\gamma} - \gamma)$ where $\tilde{\gamma}$ lies between

$\hat{\gamma}$ and γ. Since $(\hat{R}_i^* - \mathring{R}_i)\theta = -(\hat{\bar{X}}_1^* - \mu_1)\kappa$. Hence, one may rewrite the above as

$$\begin{aligned}\sqrt{N}H_{3N} &= \frac{1}{N}\sum_{i=1}^{N} -\Gamma_i\phi(R_i\gamma)R_i \cdot \sqrt{N}(\hat{\gamma}-\gamma)\mathring{R}_i\theta + \frac{1}{N}\sum_{i=1}^{N}\Gamma_i\Phi(R_i\gamma)\cdot\sqrt{N}(\hat{\bar{X}}_1^* - \mu_1)\kappa + o_p(1)\\ &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\left\{\Phi(R_i\gamma)\cdot(\hat{\bar{X}}_1^*-\mu_1)\kappa - \phi(R_i\gamma)R_i(\hat{\gamma}-\gamma)\mathring{R}_i\theta\right\} + o_p(1)\end{aligned}$$

Then,

$$\begin{aligned}\sqrt{N}(\hat{\theta}_{\text{FD}}^{\text{2S}} - \theta) &= \Omega_\Gamma^{-1}\cdot\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\Gamma_i\Big\{\mathring{R}_i\delta + (W_i^* - \Pi_i^*(\gamma))\theta + \Delta\xi_i + \Phi(R_i\gamma)(\hat{\bar{X}}_1^* - \mu_1)\kappa\\ &\quad - \phi(R_i\gamma)R_i(\hat{\gamma}-\gamma)\mathring{R}_i\theta\Big\} + o_p(1)\\ &= \Omega_\Gamma^{-1}\cdot\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\psi_i + o_p(1)\end{aligned}$$

Therefore, $\sqrt{N}(\hat{\theta}_{\text{FD}}^{\text{2S}} - \theta) \xrightarrow{d} N(0, \Omega_\Gamma^{-1}\Omega_\psi\Omega_\Gamma^{-1})$ where $\Omega_\psi = \mathbb{E}[\psi_i\psi_i']$ with

$$\begin{aligned}\sqrt{N}(\hat{\bar{X}}_1^* - \mu_1) &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\Phi(R_i\hat{\gamma})X_i}{\rho} - \mu_1\right) + o_p(1)\\ \sqrt{N}(\hat{\gamma}-\gamma) &= -\left(\mathbb{E}\left[\frac{(\partial P_i/\partial\gamma)\cdot(\partial P_i/\partial\gamma')}{P_i(1-P_i)}\right]\right)^{-1}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{D_i - P_i}{P_i(1-P_i)}\frac{\partial P_i}{\partial\gamma}\right) + o_p(1)\end{aligned}$$

Now, since

$$\sqrt{N}(\hat{\tau}_{\text{FD}}^{\text{2S}} - \tau) = \frac{1}{\mathbb{P}(D_i^* = 1)}\cdot\frac{1}{N}\sum_{i=1}^{N}\Phi(R_i\gamma)\mathring{R}_i\cdot\sqrt{N}(\hat{\theta}_{\text{FD}}^{\text{2S}} - \theta) + o_p(1) \tag{A.14}$$

Therefore,

$$\mathbb{A}\text{var}[\sqrt{N}(\hat{\tau}_{\text{FD}}^{\text{2S}} - \tau)] = \mathbb{E}(\mathring{R}_i|D_i^* = 1)\cdot\mathbb{A}\text{var}[\sqrt{N}(\hat{\theta}_{\text{FD}}^{\text{2S}} - \theta)]\cdot\mathbb{E}(\mathring{R}_i|D_i^* = 1)' \tag{A.15}$$

Asymptotic Normality of $\hat{\theta}_{\mathbf{2S}}^{\mathbf{POLS}}$: We know that

$$\sqrt{N}(\hat{\pi}_{\text{2S}}^{\text{POLS}} - \pi_2) = \left(\frac{\hat{W}_1^{*\prime}\hat{M}_1^*\hat{W}_1^*}{N}\right)^{-1}\frac{\hat{W}_1^{*\prime}\hat{M}_1^*}{\sqrt{N}}\left\{(\mathring{R}_1 - \hat{R}_1^*)\pi_1 + (W_1^* - \hat{W}_1^*)\pi_2 + \xi^1\right\}$$

Then, in the manner of the two-period panel,

$$\begin{aligned}&\frac{\hat{W}_1^{*\prime}\hat{M}_1^*\hat{W}_1^*}{N}\\ &= \frac{1}{N}\sum_{i=1}^{N}T_i\Phi^2(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^* - \left(\frac{1}{N}\sum_{i=1}^{N}T_i\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*\right)\left[\frac{1}{N}\sum_{i=1}^{N}T_i\hat{R}_i^{*\prime}\hat{R}_i^*\right]^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}T_i\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*\right)\\ &= \frac{1}{N}\sum_{i=1}^{N}\hat{\Gamma}_{1i}\hat{\Gamma}_{1i}' \xrightarrow{p} \mathbb{E}[\Gamma_{1i}\Gamma_{1i}'] = \lambda\cdot\Omega_\Gamma\end{aligned}$$

where $\hat{\Gamma}_{1i} = T_i\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime} - \left(\frac{1}{N}\sum_{i=1}^{N} T_i\Phi(R_i\hat{\gamma})\hat{R}_i^{*\prime}\hat{R}_i^*\right)\left(\frac{1}{N}\sum_{i=1}^{N} T_i\hat{R}_i^{*\prime}\hat{R}_i^*\right)^{-1} T_i\hat{R}_i^{*\prime}$ and $\Gamma_{1i} = T_i\cdot\Gamma_i = T_i\Phi(R_i\gamma)\mathring{R}_i' - \mathbb{E}[\Phi(R_i\gamma)\mathring{R}_i'\mathring{R}_i]\mathbb{E}[\mathring{R}_i'\mathring{R}_i]^{-1}T_i\mathring{R}_i'$. Therefore, using the above we can write

$$\sqrt{N}(\hat{\pi}_2^{2S} - \pi_2) = \lambda^{-1}\Omega_\Gamma^{-1}\left(\frac{\hat{W}_1^{*\prime}\hat{M}_1^*}{\sqrt{N}}\left\{(\mathring{R}_1 - \hat{R}_1^*)\pi_1 + (W_1^* - \hat{W}_1^*)\pi_2 + \xi^1\right\}\right) + o_p(1) \qquad \text{(A.16)}$$

Define $\Pi_1(\gamma) = (\Pi_{i1}(\gamma)', \ldots, \Pi_{iN}(\gamma)')'$ where $\Pi_{i1}^*(\gamma) = T_i\cdot\Pi_i^*(\gamma)$. We can express the terms inside the brackets above as

$$\begin{aligned}
&= \frac{\hat{W}_1^{*\prime}\hat{M}_1^*}{\sqrt{N}}\left\{(\mathring{R}_1 - \hat{R}_1^*)\pi_1 + (W_1^* - \hat{W}_1^* + \Pi_1^*(\gamma) - \Pi_1^*(\gamma))\pi_2 + \xi^1\right\}\\
&= \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\mathring{R}_1\pi_1}{\sqrt{N}} + \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\{(W_1^* - \Pi_1^*(\gamma))\pi_2 + \xi^1\}}{\sqrt{N}} + \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\{\Pi_1^*(\gamma) - \hat{W}_1^*\}\pi_2}{\sqrt{N}}\\
&= \sqrt{N}H_{1N}^\pi + \sqrt{N}H_{2N}^\pi + \sqrt{N}H_{3N}^\pi \qquad \text{(A.17)}
\end{aligned}$$

where

$$\sqrt{N}H_{1N}^\pi = \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\mathring{R}_1\pi_1}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\hat{\Gamma}_i\mathring{R}_i\pi_1 = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\mathring{R}_i\pi_1 + o_p(1)$$

$$\sqrt{N}H_{2N}^\pi = \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\{(W_1^* - \Pi_1^*(\gamma))\pi_2 + \xi^1\}}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\{(W_i^* - \Pi_i^*(\gamma))\pi_2 + \xi_{i1}\} + o_p(1)$$

and

$$\begin{aligned}
\sqrt{N}H_{3N}^\pi &= \frac{\hat{W}_1^{*\prime}\hat{M}_1^*\{\Pi_1^*(\gamma) - \hat{W}_1^*\}\pi_2}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\{\Pi_i^*(\gamma) - \hat{W}_i^*\}\pi_2 + o_p(1)\\
&= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\left\{\sqrt{N}(\Phi(R_i\gamma) - \hat{D}_i^*)\mathring{R}_i - \Phi(R_i\gamma)\cdot\sqrt{N}(\hat{R}_i^* - \mathring{R}_i)\right\}\pi_2 + o_p(1)\\
&= \frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\left\{\Phi(R_i\gamma)\cdot(\hat{\mathring{X}}_1^* - \mu_1)(\eta_{03} + \kappa) - \phi(R_i\gamma)R_i(\hat{\gamma} - \gamma)\mathring{R}_i(\eta_{01} + \tau)\right\} + o_p(1)
\end{aligned}$$

Hence,

$$\begin{aligned}
\sqrt{N}(\hat{\pi}_2^{2S} - \pi_2) &= \lambda^{-1}\Omega_\Gamma^{-1}\frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\Gamma_i\bigg\{\mathring{R}_i\pi_1 + (W_i^* - \Pi_i^*(\gamma))\pi_2 + \xi_{i1} + \Phi(R_i\gamma)(\hat{\mathring{X}}_1^* - \mu_1)(\eta_{03} + \kappa)\\
&\qquad - \phi(R_i\gamma)R_i(\hat{\gamma} - \gamma)\mathring{R}_i(\eta_{01} + \tau)\bigg\} + o_p(1) \qquad \text{(A.18)}\\
&= \lambda^{-1}\Omega_\Gamma^{-1}\frac{1}{\sqrt{N}}\sum_{i=1}^{N} T_i\psi_{i\pi} + o_p(1) \qquad \text{(A.19)}
\end{aligned}$$

implies that

$$\sqrt{N}(\hat{\pi}_2^{2S} - \pi_2) \xrightarrow{d} N\left(0, \frac{\Omega_\Gamma^{-1}\Omega_{\psi_\pi}\Omega_\Gamma^{-1}}{\lambda}\right) \qquad \text{(A.20)}$$

where $\Omega_{\psi_\pi} = \mathbb{E}[\psi_{i\pi}\psi'_{i\pi}]$. Similarly,

$$\begin{aligned}\sqrt{N}(\hat{\eta}_2^{\text{2S}} - \eta_2) &= (1-\lambda)^{-1}\Omega_\Gamma^{-1}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(1-T_i)\Gamma_i\Big\{\mathring{R}_i\eta_1 + (W_i^* - \Pi_i^*(\gamma))\eta_2 + \xi_{i1} \\ &\quad + \Phi(R_i\gamma)(\hat{\mathring{X}}_1^* - \mu_1)\eta_{03} - \phi(R_i\gamma)R_i(\hat{\gamma}-\gamma)\mathring{R}_i\eta_{01}\Big\} + o_p(1) &\text{(A.21)} \\ &= (1-\lambda)^{-1}\Omega_\Gamma^{-1}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(1-T_i)\psi_{i\eta} + o_p(1) &\text{(A.22)}\end{aligned}$$

which implies that

$$\sqrt{N}(\hat{\eta}_2^{\text{2S}} - \eta_2) \xrightarrow{d} N\left(0, \frac{\Omega_\Gamma^{-1}\Omega_{\psi_\eta}\Omega_\Gamma^{-1}}{(1-\lambda)}\right) \tag{A.23}$$

where $\Omega_{\psi_\eta} = \mathbb{E}[\psi_{i\eta}\psi'_{i\eta}]$. Finally, combining results in A.20 and A.23, we have

$$\mathbb{A}\text{var}[\sqrt{N}(\hat{\theta}_{\text{POLS}}^{\text{2S}} - \theta)] = \Omega_\Gamma^{-1}\left(\frac{\Omega_{\psi_\pi}}{\lambda} + \frac{\Omega_{\psi_\eta}}{(1-\lambda)}\right)\Omega_\Gamma^{-1} \tag{A.24}$$

Note that the covariance between the terms, $\sqrt{N}(\hat{\pi}_2^{\text{2S}} - \pi_2)$ and $\sqrt{N}(\hat{\eta}_2^{\text{2S}} - \eta_2)$ is zero since $T_i(1-T_i) = 0$. Finally, we can recover the asymptotic distribution of $\hat{\tau}_{\text{POLS}}^{\text{2S}}$ in a similar manner to equations A.14 and A.15. □

Proof of Theorem 2.

Bias of $\hat{\theta}_{\text{FD}}$: In matrix notation, replacing D in place of D^* in equation (6), the feasible regression is

$$\Delta Y = \ddot{R}\delta + \ddot{W}\theta + \Delta\epsilon,\ \Delta\epsilon = [(\mathring{R} - \ddot{R})\delta + (W^* - \ddot{W})\theta + \Delta\xi]$$

where ΔY, $\ddot{R}$, $\ddot{W}$, and $\Delta\epsilon$ are the N-vector data matrices. Also note that, $\ddot{R}_i \equiv (1, \ddot{X}_i)$ where $\ddot{X}_i = X_i - \bar{X}_1$ with $\bar{X}_1$ being the sample mean of covariates for those misclassified as being treated $(D_i = 1)$. Similarly, $\ddot{W}_i \equiv D_i\ddot{R}_i$.

Let $\ddot{M} = I - \ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}'$ be the residual making matrix of $\ddot{R}$. Consider the following regression,

$$\ddot{M}\Delta Y = \ddot{M}\ddot{W}\theta + \ddot{M}\Delta\epsilon \tag{A.25}$$

Then,

$$\begin{aligned}\hat{\theta}_{\text{FD}} &= \left[(\ddot{M}\ddot{W})'(\ddot{M}\ddot{W})\right]^{-1}(\ddot{M}\ddot{W})'(\ddot{M}\Delta Y) \\ &= \left(\ddot{W}'\ddot{M}\ddot{W}\right)^{-1}\ddot{W}'\ddot{M}\Delta Y \\ &= \theta + \left(\ddot{W}'\ddot{M}\ddot{W}\right)^{-1}\ddot{W}'\ddot{M}\Delta\epsilon\end{aligned}$$

This implies that

$$
\begin{aligned}
\hat{\theta}_{\mathrm{FD}} - \theta &= \left(\ddot{W}'\ddot{M}\ddot{W}\right)^{-1} \ddot{W}'\ddot{M}\Delta\epsilon \\
&= \left(\frac{\ddot{W}'\ddot{M}\ddot{W}}{N}\right)^{-1} \frac{\ddot{W}'\ddot{M}\Delta\epsilon}{N} \\
&= \left(\frac{\ddot{W}'\ddot{M}\ddot{W}}{N}\right)^{-1} \frac{\ddot{W}'\ddot{M}}{N} \left\{(\mathring{R} - \ddot{R})\delta + (W^* - \ddot{W})\theta + \Delta\xi\right\} \\
&= \underbrace{\left(\frac{\ddot{W}'\ddot{M}\ddot{W}}{N}\right)^{-1}}_{①} \left\{ \underbrace{\frac{\ddot{W}'\ddot{M}(\mathring{R} - \ddot{R})\delta}{N}}_{②} + \underbrace{\frac{\ddot{W}'\ddot{M}(W^* - \ddot{W})\theta}{N}}_{③} + \underbrace{\frac{\ddot{W}'\ddot{M}\Delta\xi}{N}}_{④} \right\} \\
&= \underbrace{\left(\frac{\ddot{W}'\ddot{M}\ddot{W}}{N}\right)^{-1}}_{①} \left\{ \underbrace{\frac{\ddot{W}'\ddot{M}\mathring{R}\delta}{N}}_{②} + \underbrace{\frac{\ddot{W}'\ddot{M}(W^* - \ddot{W})\theta}{N}}_{③} + \underbrace{\frac{\ddot{W}'\ddot{M}\Delta\xi}{N}}_{④} \right\} \qquad \text{(A.26)}
\end{aligned}
$$

Let's first consider ① which is equal to

$$
\begin{aligned}
\frac{\ddot{W}'(I - \ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}')\ddot{W}}{N} &= \frac{\ddot{W}'\ddot{W}}{N} - \frac{\ddot{W}'\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}'\ddot{W}}{N} \\
&= \frac{1}{N}\sum_{i=1}^{N} \ddot{W}_i'\ddot{W}_i - \left(\frac{1}{N}\sum_{i=1}^{N} \ddot{W}_i'\ddot{R}_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} \ddot{R}_i'\ddot{R}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} \ddot{R}_i'\ddot{W}_i\right)
\end{aligned}
$$

Now since $\bar{X}_1 \xrightarrow{p} \mathbb{E}(X|D=1)$, this implies that $\ddot{R}_i \xrightarrow{p} \dot{R}_i$ and $\ddot{W}_i \xrightarrow{p} \dot{W}_i$. Therefore,

$$
① \xrightarrow{p} \left\{\mathbb{E}(\dot{W}_i'\dot{W}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)\mathbb{E}(\dot{R}_i'\dot{R}_i)^{-1}\mathbb{E}(\dot{R}_i'\dot{W}_i)\right\}^{-1} \qquad \text{(A.27)}
$$

Consider ② which is equal to

$$
\begin{aligned}
\frac{\ddot{W}'\ddot{M}\mathring{R}\delta}{N} &= \frac{\ddot{W}'(I - \ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}')\mathring{R}\delta}{N} \\
&= \frac{\ddot{W}'\mathring{R}\delta}{N} - \frac{\ddot{W}'\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}'\mathring{R}\delta}{N} \\
&= \left\{\frac{1}{N}\sum_{i=1}^{N} \ddot{W}_i'\mathring{R}_i - \left(\frac{1}{N}\sum_{i=1}^{N} \ddot{W}_i'\ddot{R}_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} \ddot{R}_i'\ddot{R}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} \ddot{R}_i'\mathring{R}_i\right)\right\}\delta \\
&\xrightarrow{p} \left\{\mathbb{E}(\dot{W}_i'\mathring{R}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}(\dot{R}_i'\mathring{R}_i)\right\}\delta \qquad \text{(A.28)}
\end{aligned}
$$

Then, consider (3) which is equal to

$$
\begin{aligned}
&\frac{\ddot{W}'(I-\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}')(W^*-\ddot{W})\theta}{N} \\
&= \left\{\frac{\ddot{W}'(W^*-\ddot{W})}{N} - \frac{\ddot{W}'\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}'(W^*-\ddot{W})}{N}\right\}\theta \\
&\xrightarrow{p} \left\{\mathbb{E}[\dot{W}_i'(W_i^*-\dot{W}_i)] - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}[\dot{R}_i'(W_i^*-\dot{W}_i)]\right\}\theta
\end{aligned} \tag{A.29}
$$

Finally, (4) is equal to

$$
\begin{aligned}
\frac{\ddot{W}'(I-\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}')\Delta\xi}{N} &= \frac{\ddot{W}'\Delta\xi}{N} - \frac{\ddot{W}'\ddot{R}(\ddot{R}'\ddot{R})^{-1}\ddot{R}'\Delta\xi}{N} \\
&\xrightarrow{p} \mathbb{E}(\dot{W}_i'\Delta\xi_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}(\dot{R}_i'\Delta\xi_i) \\
&= \mathbb{E}(\dot{W}_i'\Delta\xi_i)
\end{aligned} \tag{A.30}
$$

since $\mathbb{E}(\dot{R}_i'\Delta\xi_i) = \mathbb{E}[(1,\dot{X}_i)'\Delta\xi_i] = [\mathbb{E}(\Delta\xi_i), \mathbb{E}(\dot{X}_i\Delta\xi_i)]' = 0.$

Consider,

$$
\begin{aligned}
\mathbb{E}[\dot{W}_i'\Delta\xi_i] &= \mathbb{E}[D_i\dot{R}_i'\Delta\xi_i] \\
&= \mathbb{E}[\mathbb{1}(R_i\gamma + U_i \geq 0, Z_i\alpha + V_i \geq 0)\dot{R}_i'\Delta\xi_i] \\
&= \mathbb{E}\left[\dot{R}_i' \cdot \mathbb{E}\left(\Delta\xi_i | U_i \geq -R_i\gamma, V_i \geq -Z_i\alpha\right) \cdot \mathbb{P}(U_i \geq -R_i\gamma, V_i \geq -Z_i\alpha)\right]
\end{aligned}
$$

Then by using the general formulas for truncated normal distributions in Tallis (1961), we can reduce the above expression by integration to

$$
\begin{aligned}
&\mathbb{E}\left(\Delta\xi_i | U_i \geq -R_i\gamma, V_i \geq -Z_i\alpha\right) \cdot \mathbb{P}(U_i \geq -R_i\gamma, V_i \geq -Z_i\alpha) \\
&= \sigma\psi_v\phi(-Z_i\alpha)\Phi\left(\frac{R_i\gamma - \rho Z_i\alpha}{\sqrt{1-\rho^2}}\right)
\end{aligned}
$$

Then,

$$
\mathbb{E}[\dot{W}_i'\Delta\xi_i] = \sigma\psi_v\mathbb{E}\left[\dot{R}_i'\phi(-Z_i\alpha)\Phi\left(\frac{R_i\gamma - \rho Z_i\alpha}{\sqrt{1-\rho^2}}\right)\right] \tag{A.31}
$$

Therefore, using A.27, A.28, A.29, A.30, and A.31 in the OLS bias expression in A.26, we obtain the desired result,

$$
\text{plim}(\hat{\theta}_{\text{FD}}) - \theta = Q^{-1}(A\delta + B\theta + C) \tag{A.32}
$$

where

$$
\begin{aligned}
Q &= \mathbb{E}(\dot{W}_i'\dot{W}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)\mathbb{E}(\dot{R}_i'\dot{R}_i)^{-1}\mathbb{E}(\dot{R}_i'\dot{W}_i) \\
A &= \mathbb{E}(\dot{W}_i'\mathring{R}_i) - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}(\dot{R}_i'\mathring{R}_i) \\
B &= \mathbb{E}[\dot{W}_i'(W_i^*-\dot{W}_i)] - \mathbb{E}(\dot{W}_i'\dot{R}_i)[\mathbb{E}(\dot{R}_i'\dot{R}_i)]^{-1}\mathbb{E}[\dot{R}_i'(W_i^*-\dot{W}_i)] \\
C &= \sigma\psi_v\mathbb{E}\left[\dot{R}_i'\phi(-Z_i\alpha)\Phi\left(\frac{R_i\gamma - \rho Z_i\alpha}{\sqrt{1-\rho^2}}\right)\right]
\end{aligned}
$$

Now since, $\hat{\tau}_{\text{FD}} = \frac{1}{N_1}\sum_{i:D_i=1} \ddot{R}_i\hat{\theta}_{\text{FD}} = \frac{1}{N_1}\sum_{i=1}^{N} D_i\ddot{R}_i\hat{\theta}_{\text{FD}}$, therefore,

$$\hat{\tau}_{\text{FD}} - \tau = \frac{1}{N_1}\sum_{i=1}^{N} D_i\ddot{R}_i\hat{\theta}_{\text{FD}} - \tau = \frac{N}{N_1}\cdot\frac{1}{N}\sum_{i=1}^{N} D_i\ddot{R}_i\cdot\left(\hat{\theta}_{\text{FD}} - \theta + \theta\right) - \tau$$

Note that $\dfrac{N}{N_1} = [\mathbb{P}(D_i = 1)]^{-1} + o_p(1)$ and since $\bar{X}_1 \xrightarrow{p} \mathbb{E}[X_i|D_i = 1]$, this implies that

$$\hat{\tau}_{\text{FD}} - \tau = \{\mathbb{P}(D_i = 1)\}^{-1}\cdot\left\{\frac{1}{N}\sum_{i=1}^{N} D_i\dot{R}_i\theta + \frac{1}{N}\sum_{i=1}^{N} D_i\dot{R}_i\left(\hat{\theta}_{\text{FD}} - \theta\right)\right\} - \tau + o_p(1)$$

This implies,

$$\begin{aligned}
\text{plim}(\hat{\tau}_{\text{FD}}) - \tau &= \mathbb{E}[\dot{R}_i\theta|D_i = 1] + \mathbb{E}[\dot{R}_i|D_i = 1]\cdot\text{plim}(\hat{\theta}_{\text{FD}} - \theta) - \tau \\
&= \tau + \mathbb{E}[\dot{R}_iQ^{-1}(A\delta + B\theta + C)|D_i = 1] - \tau \\
&= \mathbb{E}[\dot{R}_iQ^{-1}(A\delta + B\theta + C)|D_i = 1]
\end{aligned}$$

Bias of $\hat{\theta}_{\text{POLS}}$: Replacing D in place of D^* in equation (7), the feasible pooled regression is

$$Y_i = \ddot{R}_i\eta_1 + \ddot{W}_i\eta_2 + T_i\cdot\ddot{R}_i\delta + T_i\cdot\ddot{W}_i\theta + \epsilon_i,\ i = 1,\dots,N \tag{A.33}$$

where $\epsilon_i = \left[(\mathring{R}_i - \ddot{R}_i)\eta_1 + (W_i^* - \ddot{W}_i)\eta_2 + T_i\cdot(\mathring{R}_i - \ddot{R}_i)\delta + T_i\cdot(W_i^* - \ddot{W}_i)\theta + \xi_i\right]$. Consider the following separate regression for the $T_i = 1$ sample,

$$Y_i \text{ on } \ddot{R}_i, \ddot{W}_i \ \text{ for } \ T_i = 1$$

where we are interested in estimating the coefficient on W_i. Let Y_1, $\ddot{R}_1$, $\ddot{W}_1$, and ϵ_1 represent the N-vector data matrices with the i-th element given by $Y_{i1} = T_iY_i$, $\ddot{R}_{i1} = T_i\ddot{R}_i$, $\ddot{W}_{i1} = T_i\ddot{W}_i$, and $\epsilon_{i1} = T_i\epsilon_i$ respectively. We can then use Frisch-Waugh to obtain

$$\ddot{M}_1Y_1 = \ddot{M}_1\ddot{W}_1\pi_2 + \ddot{M}_1\epsilon_1$$

where $\pi_2 = \eta_2 + \theta$, $\ddot{M}_1 = I_1 - \ddot{R}_1(\ddot{R}_1'\ddot{R}_1)^{-1}\ddot{R}_1'$ is the residual making matrix for $\ddot{R}_1$. Also, I_1 is the matrix with i-th diagonal elements given by T_i.

$$\begin{aligned}
\hat{\pi}_2 &= \left(\ddot{W}_1'\ddot{M}_1\ddot{W}_1\right)^{-1}\ddot{W}_1'\ddot{M}_1Y_1 \\
&= \pi_2 + \left(\ddot{W}_1'\ddot{M}_1\ddot{W}_1\right)^{-1}\ddot{W}_1'\ddot{M}_1\epsilon_1
\end{aligned}$$

which implies

$$\begin{aligned}
\hat{\pi}_2 - \pi_2 &= \left(\frac{\ddot{W}_1'\ddot{M}_1\ddot{W}_1}{N}\right)^{-1}\frac{\ddot{W}_1'\ddot{M}_1\epsilon_1}{N} \\
&= \left(\frac{\ddot{W}_1'\ddot{M}_1\ddot{W}_1}{N}\right)^{-1}\frac{\ddot{W}_1'\ddot{M}_1}{N}\{(\mathring{R}_1 - \ddot{R}_1)\pi_1 + (W_1^* - \ddot{W}_1)\pi_2 + \xi^1\} \\
&= \left(\frac{\ddot{W}_1'\ddot{M}_1\ddot{W}_1}{N}\right)^{-1}\left\{\frac{\ddot{W}_1'\ddot{M}_1\mathring{R}_1\pi_1}{N} + \frac{\ddot{W}_1'\ddot{M}_1(W_1^* - \ddot{W}_1)\pi_2}{N} + \frac{\ddot{W}_1'\ddot{M}_1\xi^1}{N}\right\}
\end{aligned} \tag{A.34}$$

where $\mathring{R}_1$, W_1^*, and ξ^1 are again data matrices with i-th element given by $\mathring{R}_{i1} = T_i\mathring{R}_i$, $W_{i1}^* = T_iW_i^*$, and $\xi_i^1 = T_i\xi_i = T_i\xi_{i1}$ respectively. Also, $\pi_1 = \eta_1 + \delta$.

Following the proof as in the case of the two period panel, we get

$$\begin{aligned}
\frac{\ddot{W}_1'\ddot{M}_1\ddot{W}_1}{N} &= \frac{\ddot{W}_1'(I_1 - \ddot{R}_1(\ddot{R}_1'\ddot{R}_1)^{-1}\ddot{R}_1')\ddot{W}_1}{N} \\
&= \frac{\ddot{W}_1'I_1\ddot{W}_1}{N} - \frac{\ddot{W}_1'\ddot{R}_1(\ddot{R}_1'\ddot{R}_1)^{-1}\ddot{R}_1'\ddot{W}_1}{N} \\
&= \frac{1}{N}\sum_{i=1}^{N} T_i\ddot{W}_i'\ddot{W}_i - \left(\frac{1}{N}\sum_{i=1}^{N} T_i\ddot{W}_i'\ddot{R}_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} T_i\ddot{R}_i'\ddot{R}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} T_i\ddot{R}_i'\ddot{W}_i\right) \\
&\xrightarrow{p} \lambda\cdot\left\{\mathbb{E}(\dot{W}_i'\dot{W}_i|T_i=1) - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=1)[\mathbb{E}(\dot{R}_i'\dot{R}_i)|T_i=1]^{-1}\mathbb{E}(\dot{R}_i'\dot{W}_i|T_i=1)\right\}
\end{aligned} \tag{A.35}$$

and,

$$\begin{aligned}
\frac{\ddot{W}_1'\ddot{M}_1\mathring{R}_1\pi_1}{N} &\xrightarrow{p} \lambda\left\{\mathbb{E}(\dot{W}_i'\mathring{R}_i|T_i=1) - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=1)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=1)]^{-1}\mathbb{E}(\dot{R}_i'\mathring{R}_i|T_i=1)\right\}\pi_1 \\
\frac{\ddot{W}_1'\ddot{M}_1(W_1^* - \ddot{W}_1)\pi_2}{N} &\xrightarrow{p} \lambda\Big\{\mathbb{E}[\dot{W}_i'(W_i^* - \dot{W}_i)|T_i=1] - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=1)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=1)]^{-1} \\
&\qquad \mathbb{E}[\dot{R}_i'(W_i^* - \dot{W}_i|T_i=1)]\Big\}\pi_2 \\
\frac{\ddot{W}_1'\ddot{M}_1\xi^1}{N} &\xrightarrow{p} \lambda\left\{\mathbb{E}(\dot{W}_i'\xi_{i1}|T_i=1) - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=1)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=1)]^{-1}\mathbb{E}(\dot{R}_i'\xi_{i1}|T_i=1)\right\} \\
&= \lambda\cdot\mathbb{E}(\dot{W}_i'\xi_{i1}|T_i=1)
\end{aligned} \tag{A.36}$$

Similarly, let Y_0, $\ddot{R}_0$, $\ddot{W}_0$, ϵ_0 be again data matrices with the i-th element given by $Y_{i0} = (1-T_i)Y_i$, $\ddot{R}_{i0} = (1-T_i)\ddot{R}_i$, $\ddot{W}_{i0} = (1-T_i)\ddot{W}_i$, and $\epsilon_{i0} = (1-T_i)\epsilon_i$, respectively. Then, using Frisch-Waugh,

$$\ddot{M}_0Y_0 = \ddot{M}_0\ddot{W}_0\eta_2 + \ddot{M}_0\epsilon_0 \tag{A.37}$$

where $\ddot{M}_0 = (I_0 - \ddot{R}_0(\ddot{R}_0'\ddot{R}_0)^{-1}\ddot{R}_0')$ is the residual making matrix for $\ddot{R}_0$. Then the bias for $\hat{\eta}_2$ is given as

$$\hat{\eta}_2 - \eta_2 = \left(\frac{\ddot{W}_0'\ddot{M}_0\ddot{W}_0}{N}\right)^{-1}\left\{\frac{\ddot{W}_0'\ddot{M}_0\mathring{R}_0\eta_1}{N} + \frac{\ddot{W}_0'\ddot{M}_0(W_0^* - \ddot{W}_0)\eta_2}{N} + \frac{\ddot{W}_0'\ddot{M}_0\xi^0}{N}\right\} \tag{A.38}$$

where $\mathring{R}_0$, W_0^*, and ξ^0 are defined analogously and for each term we have

$$\begin{aligned}
\frac{\ddot{W}_0'\ddot{M}_0\ddot{W}_0}{N} &\xrightarrow{p} (1-\lambda)\Big\{\mathbb{E}(\dot{W}_i'\dot{W}_i|T_i=0) - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=0)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=0)]^{-1}\mathbb{E}(\dot{R}_i'\dot{W}_i|T_i=0)\Big\} \\
\frac{\ddot{W}_0'\ddot{M}_0\mathring{R}_0\eta_1}{N} &\xrightarrow{p} (1-\lambda)\left\{\mathbb{E}(\dot{W}_i'\mathring{R}_i|T_i=0) - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=0)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=0)]^{-1}\mathbb{E}(\dot{R}_i'\mathring{R}_i|T_i=0)\right\}\eta_1 \\
\frac{\ddot{W}_0'\ddot{M}_0(W_0^*-\ddot{W}_0)\eta_2}{N} &\xrightarrow{p} (1-\lambda)\Big\{\mathbb{E}[\dot{W}_i'(W_i^*-\dot{W}_i|T_i=0)] - \mathbb{E}(\dot{W}_i'\dot{R}_i|T_i=0)[\mathbb{E}(\dot{R}_i'\dot{R}_i|T_i=0)]^{-1} \\
&\qquad \mathbb{E}[\dot{R}_i'(W_i^*-\dot{W}_i|T_i=0)]\Big\}\eta_2 \\
\frac{\ddot{W}_0'\ddot{M}_0\xi^0}{N} &\xrightarrow{p} (1-\lambda)\,\mathbb{E}(\dot{W}_i'\xi_{i0}|T_i=0)
\end{aligned} \tag{A.39}$$

Then together with equations A.34, A.36, A.38, and A.39 we obtain

$$\begin{aligned}
\text{Bias}(\hat{\theta}_{\text{POLS}}) &= \text{Bias}(\hat{\pi}_2) - \text{Bias}(\hat{\eta}_2) \\
&= Q_1^{-1}\left(A_1\pi_1 + B_1\pi_2 + C_1\right) - Q_0^{-1}\left(A_0\eta_1 + B_0\eta_2 + C_0\right)
\end{aligned}$$

where all the above objects are defined analogously to the case for $\hat{\theta}_{\text{FD}}$. The asymptotic bias expression for $\hat{\tau}_{\text{POLS}}$ can be derived analogously to the case for $\hat{\tau}_{\text{FD}}$. □

B Empirical Application: Additional Details

B.1 National Old Age Pension Scheme and Sample Construction

The first iteration of National Old Age Pension Scheme (NOAPS) began in 1995 and was targeted towards people aged 65 or above. Under the program, eligible individuals received a small monthly pension of INR 75 (around 2$). However, this version of NOAPS was relatively unsuccessful in targeting the elderly (Kaushal (2014); Asri (2019)). This has been attributed to a meager pension amount, an arbitrary targeting rule, and only partial coverage of the elderly poor population in the country (Asri (2019); Alam (2004)). Since then the program has undergone several iterations, the first being in 2007 when it was renamed the IGNOAPS. In this revision, the targeting rule was changed to include old individuals living below the official poverty line. The pension benefits were also increased to INR 200 (around 5$). The final revision of the targeting criteria was done in 2011, where the age for being eligible was lowered from 65 to 60 (Unnikrishnan and Imai (2020)).

Our sample for the analysis is constructed by first removing households from both survey rounds where no anthropometric measurements were taken for children below 11 years of age. Second, we also remove households that received old-age pension benefits in 2005, before revisions to the targeting criteria. Table C.1 presents the summary statistics on key variables of interest for the two time periods.

B.1.1 Descriptive Summary

The revision in the targeting rule clearly had an impact, as the program's coverage more than doubled in 2012. This is consistent with other studies that find improved coverage due to the revised targeting criteria (Asri (2019)). Figure C.1 shows the age-wise distribution of the proportion of individuals reporting having received pensions under IGNOAPS. While the self-reported program status may be under-reported, we observe that a very small proportion of individuals below the age of 60 report having received benefits under the IGNOAPS.

Figure C.2 shows the trends in the prevalence of child underweight and stunting for self-reported IGNOAPS pension benefit status. The proportion of underweight children in the treated group remains unchanged but shows a decline in the control group. This could be driven by the under-reporting of IGNOAPS pension benefits as some of the households actually receiving benefits could be in the control group. Stunting shows a decline in both the treated and control groups.

B.2 Public Distribution System and Sample Construction

Public Distribution System is the world's largest food safety net program that distributes highly subsidized food grains, mainly rice and wheat, to close to a billion people in 180 million poor eligible households Balani (2013). Initially, the program had universal coverage, but in 1997 was transformed into TPDS, which emphasized targeted food subsidies for only the poor eligible households. The task of identifying households falls on state governments who, in turn, rely on the elected village governments to identify the poor.

In general, three types of ration cards are issued to beneficiary households. The APL (for households with income above poverty line) ration card, the BPL (for households with income below poverty line) ration card, and the AAY (Antyodaya Anna Yojana) for the poorest households. The BPL and the AAY ration card owners are the primary beneficiaries of the PDS subsidy with AAY households getting a higher subsidy. The APL ration card families can buy food grains from ration shops at a modest subsidy which depends on availability of grains after allocation of the BPL households (Ministry of Consumer Affairs, Food and Public Distribution (2013); Kaushal and Muchomba (2015)).

We categorize households with no ration card or APL (above poverty line) card as the control group as they are not the main beneficiaries of TPDS. The treated group comprises of households owning a BPL or an AAY ration card, as they are the observed beneficiaries of the TPDS program. AAY ration card is issued to the poorest households within the BPL category with more generous subsidies. Since the treatment group has to be the same in both survey rounds, we retain only those households in the final sample whose ration card status has remained unchanged in both periods. We also remove households in the state of Tamil Nadu as it followed universal PDS and not TPDS during our period of analysis.

B.2.1 Descriptive Summary

Table C.2 presents the summary statistics on key variables of interest from the baseline and endline IHDS. In terms of income and assets, the treated households are poorer than control households.[1] Interestingly, treated households also report having higher memberships in local caste associations and having links with local politicians relative to the control group.

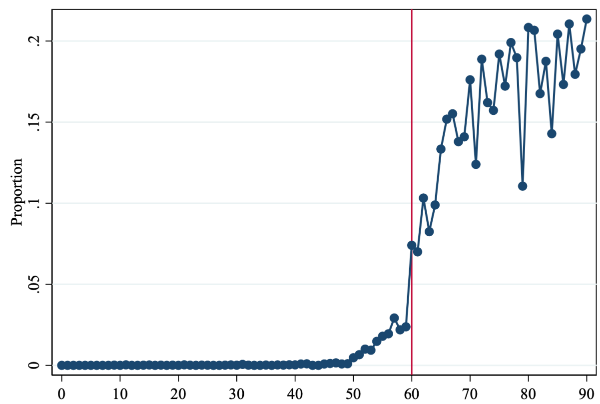
[1]This is to be expected as BPL ration cards are targeted towards poor households.

In particular, figure C.3 shows that the market price for both commodities increased whereas PDS prices registered a marginal decline between baseline and endline IHDS. Figure C.4 plots the trends in the average per-capita calorie intake (outcome) of the treated and control households. We see that the calorie trend of the treated group is stable between baseline and endline whereas the same for control households shows a marked decline. This, however, would likely be exaggerated if there is high exclusion of BPL-eligible households from the treatment group.

To see whether our proposed instruments are correlated with S, we generate an artificial S as a product of the poverty status of the households and the BPL dummy (see Table C.4). Poverty status is determined based on the principal component of all the household characteristics which were used by state officials to target households for a BPL ration card. These are dummies for small landowners, below the official poverty line, caste status, rural regions, vehicle ownership, color TV ownership, cooler ownership, electric fan, refrigerator, kaccha house, LPG gas connection, and tractor/thresher ownership. Households are coded as poor if the principal component based index of all these variables is below the sample average, the idea being that these households should have been allocated the BPL ration card.

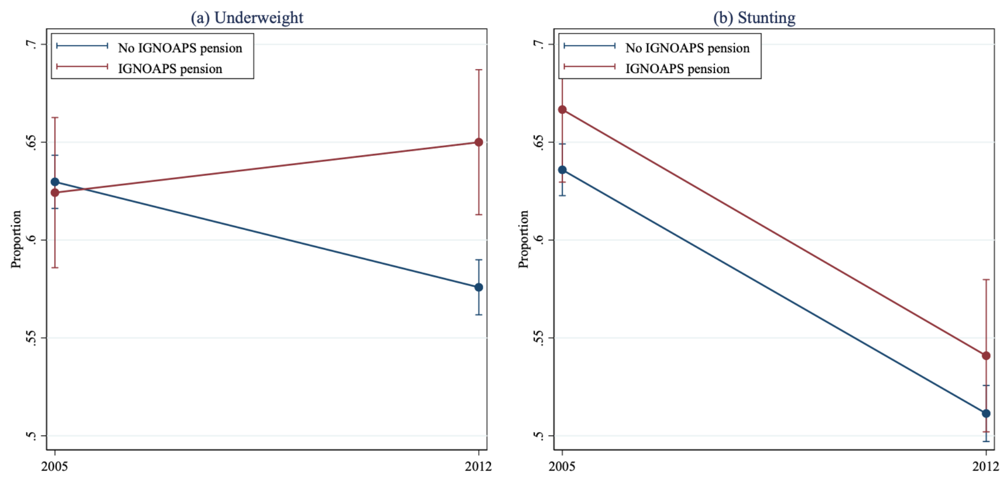
C Figures and Tables

Figure C.1: Age-wise Proportion of Individuals Reporting Receiving Benefits Under the IG-NOAPS



Note: The red vertical line represents the revision in the IG-NOAPS targeting rule of reducing the elegibility age from 65 to 60 years between 2005 and 2012.

Figure C.2: Underweight and Stunting Trends in Self-Reported IGNOAPS Status between Baseline and Endline



Note: This figure shows the proportion of households in the baseline and endline IHDS which have at least one underweight and stunted children within 2 to 11 years. A child is stunted if the height-for-age z-score (HAZ) is less than -2. A child is underweight if the weight-for-age z-score (WAZ) is less than -2.

Figure C.3: Market and PDS Price of Rice and Wheat

Note: This figure plots the market and PDS price of rice and wheat between baseline and endline with 95% confidence intervals. These are nominal prices.

Table C.1: Summary Statistics

	Baseline (2005)		Endline (2012)	
	Control (No pension)	Treated (Pension)	Control (No pension)	Treated (Pension)
Households	6498	660	6498	660
Outcomes				
Stunted (HAZ <- 2)	0.64	0.67	0.51	0.54
Underweight (WAZ < -2)	0.63	0.62	0.58	0.65
Covariates				
Average age of surveyed children (years)	5.53	5.59	7.80	7.49
Operated land (ha)	0.09	0.13	0.12	0.24
Number of family members	6.91	8.18	7.46	9.28
Children under 15 (no)	3.18	3.31	3.15	3.66
Adults members over 21 (no)	3.20	4.25	3.50	4.82
Old members over 60 (no)	0.37	0.88	0.50	1.42
Any adult illiterate	0.76	0.80	0.80	0.86
Monthly cons. expenditure (rs/person)	1194.88	1007.39	1603.70	1380.05
Assets (no)	10.62	9.79	16.91	19.45
Any member in MGNREGA work	0.00	0.00	0.37	0.44
Has BPL ration card	0.30	0.43	0.41	0.63
Religion: muslim	0.16	0.09		
Religion: others	0.02	0.02		
Caste: scheduled caste and scheduled tribes	0.31	0.34		
Caste: other backward caste	0.34	0.42		
Predictor of S				
Non-household member present during survey				0.49

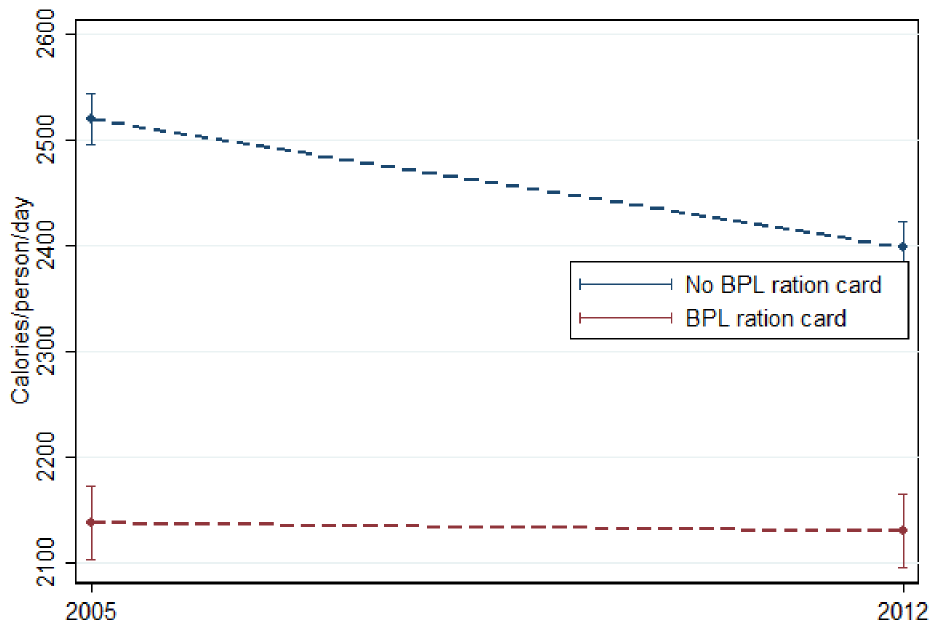
Notes: A child is stunted if the height for age z-score (HAZ) is less than -2. A child is underweight if the weight for age z-score (WAZ) is less than -2. BPL ration cards are given to poor households below the official poverty line. Households with BPL cards can access foodgrains at highly subsidized prices. MGNREGA stands for the Mahatma Gandhi Rural Employment Guarantee Scheme. This is India's large-scale anti-poverty rural workfare program. It was introduced in 2005 and provides 100 days per year of voluntary employment at minimum wages to individuals in the working age group. The MGNREGA is mostly operational in rural areas and provides unskilled labor employment on local public work projects. Scheduled castes and scheduled tribes are considered to be disadvantaged social groups in India.

Table C.2: Summary Statistics

Variables	Baseline (2005)		Endline (2012)	
	Control (No BPL card)	Treated (BPL card)	Control (No BPL card)	Treated (BPL card)
Households	6,345	3,035	6,345	3,035
Outcome				
Calories consumed per person per day	2519.83	2137.82	2398.78	2129.91
Covariates				
Operated land (ha)	2.04	0.99	1.36	0.82
Number of family members	6.92	5.87	5.40	4.97
Monthly cons. expenditure (rs/person)	1761.93	1079.49	2336.43	1521.29
Assets: owns any vehicle	0.70	0.57	0.70	0.60
Assets: owns motor vehicle	0.24	0.03	0.37	0.13
Assets: owns cooler or AC	0.14	0.01	0.21	0.04
Assets: owns TV	0.58	0.25	0.65	0.47
Assets: own electric fan	0.66	0.32	0.77	0.54
Assets: own refrigerator	0.17	0.01	0.31	0.04
Permanent house structure	0.66	0.36	0.75	0.46
Household has electricity	0.76	0.62	0.86	0.80
Household owns farm equipment	0.13	0.01	0.11	0.01
Any member in MGNREGA work	0.00	0.02	0.15	0.26
Benefits from govt. programs (000 rupees)	0.01	0.01	0.02	0.04
Religion: hindu	0.83	0.90	-	-
Religion: muslim	0.10	0.05	-	-
Religion: others	0.05	0.01	-	-
Caste: scheduled caste and scheduled tribes	0.17	0.41	-	-
Caste: other backward caste	0.35	0.41	-	-
Caste: general	0.32	0.12	-	-
Proportion of area under rice-wheat	0.46	0.45	-	-
Proportion in rural areas	0.95	0.98	-	-
Predictors of S				
Household member(s) in local caste association	0.10	0.17	-	-
Household member(s) links to local officials	0.12	0.14	-	-

Notes: MGNREGA stands for the Mahatma Gandhi Rural Employment Guarantee Scheme. MGNREGA is India's large-scale anti-poverty rural workfare program. It was introduced in 2005 and provides 100 days per year of voluntary employment at minimum wages to individuals in the working age group. The MGNREGA is mostly operational in rural areas and provides unskilled labor employment on local public work projects. Benefit from government programs is calculated as the total transfers received from scholarships, old age pension, maternity scheme, disability scheme, income generation programs other than MGNREGA, assistance from drought/flood compensation, and insurance payouts.

Figure C.4: Average Calorie Trends for Treated and Control Households Between Baseline and Endline



Note: This figure presents the average trends in the per-capita calorie consumed per-day for BPL ration card and non-BPL ration card owning households between the baseline and the endline samples with 95% confidence intervals.

Table C.3: Cross Tabulation of Poverty Status and Ownership of a BPL Ration Card

	Non-BPL ration card	BPL ration card	Total
	Control	Treated	
Non-poor	2,731	226	2,957
	(92.4)	(7.6)	(100)
Poor	3,614	2,809	6,423
	(56.3)	(43.7)	(100)
Total	6,345	3,035	9,380
	(67.6)	(32.4)	(100)

Notes: Numbers in parenthesis are row percentages. We label a household as being poor based on the principal component of all the household characteristics which state officials used to target households for a BPL ration card. These are dummies for small landowners, below official poverty line status, caste status, rural region, vehicle ownership, color TV ownership, cooler ownership, electric fan, refrigerator, kaccha house, LPG gas connection, and tractor/thresher ownership. Households are coded as poor if the principal component based index of all these variables is below the sample average. These are dummies for small landowners, below the official poverty line, caste status, rural regions, vehicle ownership, color TV ownership, cooler ownership, electric fan, refrigerator, kaccha house, LPG gas connection, and tractor/thresher ownership. Households are coded as poor if the principal component based index of all these variables is below the sample average.

Table C.4: Correlation between the Instruments and Generated S

	(1)	(2)
Dependent variable: S =Poor status$\times$BPL card ownership		
Any member in local caste associations	0.387***	0.481***
	(0.066)	(0.074)
Panchayat official close to household	0.116*	0.230***
	(0.061)	(0.067)
Log operated land		-0.478***
		(0.060)
Log consumption expenditure		-0.251***
		(0.045)
Any member literate dummy		-0.142***
		(0.053)
Hindu dummy		0.510***
		(0.099)
Other caste dummy		-0.708***
		(0.075)
Other backward caste dummy		-0.152**
		(0.061)
Rural dummy		0.365***
		(0.128)
Any vehicle owned dummy		-0.191***
		(0.049)
Motor vehicle owned dummy		-0.959***
		(0.104)
Cooler owned dummy		-1.220***
		(0.195)
TV owned dummy		-0.375***
		(0.051)
Electric fan owned dummy		-0.378***
		(0.059)
Refrigerator owned dummy		-1.146***
		(0.190)
Kaccha house dummy		-0.424***
		(0.049)
LPG gas dummy		0.346***
		(0.064)
Tractor/thresher owned dummy		-1.027***
		(0.146)
Constant	-0.591***	1.545***
	(0.034)	(0.349)
N	9380	9380

Notes: Columns (1) and (2) report estimates across two different specifications; the first regresses instruments on S whereas the second controls for additional covariates. S is defined as the product of poverty status and BPL ration card owning status. Poverty status is based on the principal component of all the household characteristics which were used by state officials to target households for a BPL ration card. These are dummies for small landowners, below the official poverty line, caste status, rural regions, vehicle ownership, color TV ownership, cooler ownership, electric fan, refrigerator, kaccha house, LPG gas connection, and tractor/thresher ownership. Households are coded as poor if the principal component based index of all these variables is below the sample average. Standard errors in parentheses clustered at the village level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.