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Abstract

We provide a model of individual choice in which the decision maker is constrained and chooses from a subset of the available alternatives given a set of *attributes*. We introduce an attribute competition filter which provides conditions under which an alternative continues to be considered from a subset of alternatives and a subset of attributes. We use two axioms to characterize a rational choice function from the consideration sets, *Single Reversal in Attributes (SRA)* and *Contraction Consistency with Fixed Attributes (CCFA)*. The former only allows for a single reversal in choice from a subset of the attributes, while the latter requires choices to be contraction consistent. We show that a choice function from consideration sets under attributes is rationalizable if and only if the choice function satisfies SRA and CCFA. In another section, we consider the dual problem: The alternatives considered are exogenously visible i.e. all the alternatives are considered and limited attention is paid to the attributes available while the preference relation is over the set of alternatives via individual attributes.

JEL classification: D00, D01

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1 Introduction

There is ample evidence from experiments on consumer behavior to suggest that consumers do not take into consideration all the available alternatives and that the cognitive burden increases with the number of available alternatives.[1] In standard rational choice theory, having more alternatives is beneficial since better alternatives may be available. However, when the best alternative is available, adding more alternatives may create a 'attention overload' due to which the decision-maker (DM) may fail to observe the best alternative. This may lead to sub-optimal choices. When alternatives have different attributes which are visible to the DM, the consideration of a set of alternatives may depend on the attributes which were considered during the decision-making process. In this paper, we provide a model of individual choice from different menus over feasible alternatives in the context of a set of attributes where the limited consideration is driven by the attention paid to different attributes.

There are many settings where only a subset of the alternatives may be considered while choosing from a set of alternatives and attributes. We provide some examples:

1. A DM wants to buy a box of cereals with a certain attributes. If there are too many options and too many attributes under consideration, it is possible that the DM may not notice the 'best' alternative in the set of available options. With fewer alternatives consisting of the 'best' alternative and a subset of the attributes, it is more likely that the best alternative is considered and chosen.
2. A DM wants to buy a car. If she considers too many attributes like mileage, utility, additional features and design, then too many options may lead to sub-optimal choice. By narrowing down the criteria for choosing the car, she may be able to choose the best car from the set.

Note that in the above examples, considering a product is not the same as buying a product. Therefore, the size of the feasible set of alternatives and the set of attributes will affect the consideration set of the DM. We introduce the concept of competition filter which formally captures the notion of a consideration given a set of alternatives and set of attributes. We define *Attribute Competition Filter (ACF)* which is based on the notion of a competition filter used in Lleras et al. (2017). An ACF requires that (i) if an alternative is considered from a set of alternatives given a set of attributes, then it should continue to be considered from a subset of the initial set of alternatives given a subset of the initial set of attributes, (ii) When the attribute set is fixed, the alternatives considered in a subset of the initial superset of alternatives must be all the alternatives that are in the referred subset of alternatives and which were

[1] See Huettner et al. (2019) and Zhong (2022) for example.

considered in the initial superset of alternatives, and (iii) When the feasible set of alternatives is a singleton, then that alternative is considered no matter what the set of attributes corresponds to it.

The objective of the paper is to characterize conditions under which the competition filters allow for the rationalizability of the choice function given the menu of alternatives and the set of attributes. We say that a choice function is an *Overwhelming Choice with Attributes (OCA)* if it can be rationalized by a linear order which picks the highest ranked alternative from the consideration set for a given menu and set of attributes. It is well-known that a choice function is rationalizable if and only if it satisfies the Weak Axiom of Revealed Preference (WARP) (Sen (1971)). However, this result assumes that the DM pays attention to all the alternatives. Our paper attempts to answer the same question in a setting where the DM is attention constrained and does not consider all the alternatives in the presence of different attributes.

We identify two conditions which are necessary and sufficient for the choice function to be an OCA. The first, *Single Reversal in Attributes (SRA)* requires that if the choice changes from x being chosen initially to y while choosing from a subset of the attributes, while the menu of alternatives is held fixed in which both x and y are available, then the choice from the further subset of the attributes given the same menu cannot be x. In other words the choice can 'reverse' only once for any pair of alternatives. This axiom is related to WARP and Weak WARP as in Manzini and Mariotti (2007). But they work on the restrictions of the choice function when the alternative sets are varied. Here, in contrast, the axiom is applicable when the alternative set is fixed and the attribute set is varied. As an illustration, suppose $S = \{x, y, z\}$ and $A = \{a, b, c\}$. A choice function where $c(S, A) = \{x\}, c(S, \{a, b\}) = \{y\}$ and $c(S, \{a\}) = \{x\}$ violates SRA since there are two choice reversals involving x and y. A violation of SRA is seen as 'irrational' since there is no reason for DM to switch choices twice when attribute sets contract given that she has already paid attention to the superior alternative when a bigger set of attributes is available. Therefore, SRA can also be interpreted as the DM being reasonably attentive.

The second axiom is a contraction consistency condition with fixed attributes (CCFA). It states that when the attribute set is fixed, an alternative should continue to be chosen from any subset of the alternatives which includes that alternative. For example, suppose $X = \{x, y, z\}$ and $A = \{a, b, c\}$. A choice function where $c(X, A) = \{x\}$, $c(\{x, y\}, A) = \{y\}$ violates CCFA. Note that the two axioms are logically independent in the two examples presented above.[2]

Our main result states that a choice function is rationalizable or an OCA if and

[2]Example 1 in Section 2 illustrates the violation of CCFA. We provide a proof of the independence of the two axioms in Section 3.

only if it satisfies SRA and CCFA. The proof of the main result proceeds by first deriving the preference relation over the set of alternatives. Here, SRA is used to prove the completeness of the preference relation while both SRA and CCFA are used in proving the transitivity of the preference relation. The second part of proving that the choice function is an OCA is to show that there is an Attribute Competition Filter (ACF) that is being used to rationalize the choice over the given menu and the set of attributes. CCFA is used frequently to show that the above claim is true.

In another section we consider the case where all the alternatives are considered, while the attention paid to different attributes is limited and may depend on the number of alternatives in the menu. This can be interpreted as the dual of the original problem: all the alternatives are considered and consideration sets are subsets of the attributes, i.e., not all the attributes are always considered. When more alternatives are present, they may distract the DM leading to limited attention on the attributes. We introduce the problem of identifying the conditions under which the choice function can be rationalized in this setting. We provide an example of a constrained maximin choice function in which first the set of attributes considered are fixed upon. Then each alternative is mapped to the least payoff (or utility) it generates across all the considered attributes (according to the first step). Finally, the alternative which generates the maximum payoff among the least payoffs (corresponding to the previous step) is chosen.

There are many papers on revealed preference theory, e.g., Samuelson (1948), Richter (1966), Houthakker (1950), and Nishimura et al. (2017), which explore conditions for revealed preference identification in a cardinal setting with prices and incomes. Others like, Echenique and Saito (2019), Masatlioglu et al. (2012) and Lleras et al. (2017) take the axiomatic approach in characterizing rationalizable choice functions in different settings. Attributes were first introduced to the economics literature by Lancaster (1966). Manzini and Mariotti (2007) provides a shortlisting method via which choices are made. Cherepanov et al. (2013) uses a psychological rationalizability procedure (which maybe interpreted as attributes) via which choices are made. They use a different procedure where there is an attribute or rationale which can rationalize the choice. In a stochastic choice framework, Manzini and Mariotti (2014) works on limited attention with consideration probabilities of various alternatives which are used to construct a stochastic choice rule. There several other papers which deal with limited attention in different settings: Matějka and McKay (2015), Cattaneo et al. (2020), Dean et al. (2017), Ahumada and Ülkü (2018) etcetera. See Caplin (2016) for a detailed review.

Masatlioglu et al. (2012) and Lleras et al. (2017) characterize conditions for rationalizability of choice functions in the presence of limited attention by introducing attention filters which specify the consideration sets. Both these works identify a variant of the Weak Axiom of Revealed Preference with Limited Attention. These works also indicate that broader conditions are needed to rationalize a choice function with consideration sets and limited attention. Our work adds to this stream of literature by characterizing conditions for rationalizability in the presence of different sets of attributes. Lleras et al. (2017) use a transitive closure to obtain a linear order, while we have an explicit linear order. The proof of our main theorem uses a recursive construction on the attribute competition filter for the sufficiency part of the proof.

Our paper identifies a similar condition as the ones in Masatlioglu et al. (2012) and Lleras et al. (2017) where the choice reversals allow for the preference to be identified. Our paper differs from these in terms of the context due to the presence of attributes. Even though SRA is similar to the limited attention based WARP in the literature on revealed preference, this axiom along with CCFA are identified as necessary and sufficient for rationalizability of choice with attributes and limited attention.

Another paper that is related to ours is Kimya (2018). It considers model of consideration set formation in the presence of attributes and limited attention. They allow choice reversals due to attraction and compromise effects. However, their model is based ordering the set of attributes and then using thresholds to characterize the choice behavior.

We present the model in Section 2 where we set up the choice problem. Section 3 presents the Axioms. Section 4 provides the statement and proof of the main result on rationalizability of choice with attributes. Section 5 sheds light on the dual of the problem with limited attention on the set of attributes. We conclude in Section 6.

2 Model

Let $\mathbf{X}$ be the finite set of all alternatives and $\mathcal{X}$ be the set of all non-empty subsets of $\mathbf{X}$. Let $\mathbf{A}$ be the set of all attributes and $\mathcal{A}$ be the set of all subsets of $\mathbf{A}$ such that (i) $B \in \mathcal{A} \implies C \in \mathcal{A}, \forall C \subseteq B$, (ii) $C, D \in \mathcal{A} \implies C \cap D \in \mathcal{A}$, and (iii) $\mathbf{A} \in \mathcal{A}$. Note that we are allowing the empty set of attributes to be a part of $\mathcal{A}$. This is easy to motivate: If the tags (like cost and rating) on an online shopping search engine are interpreted as attributes then an empty set of attributes corresponds simply to the data collected on choices made without shopping tags. An alternative interpretation is that situations with empty set of attributes is the default situation where no particular attribute is considered for decision-making.

We can observe $S(\in \mathcal{X})$, the menu of alternatives and $A(\in \mathcal{A})$, the set of relevant attributes in any choice situation. The DM does not consider all the alternatives in a menu given a set of attributes. We introduce the notion of competition filters to formalize the notion of 'consideration'. A function $\Gamma : \mathcal{X} \times \mathcal{A} \to \mathcal{X}$ is a *competition filter* if $\Gamma(V, D) \subseteq V, \forall V \in \mathcal{X}, \forall D \in \mathcal{A}$.

Definition 1 $\Gamma : \mathcal{X} \times \mathcal{A} \to \mathcal{X}$ *is said to be an Attribute Competition Filter* ***(ACF)*** *if for all* $y \in \mathbf{X}$*,* $A, B \in \mathcal{A}$*, such that* $B \subseteq A$*, for all* $T, S \in \mathcal{X}$ *such that* $T \subseteq S$ *and* $|T| \geq 2$*,*

(i) $[x \in T \subseteq S,\ x \in \Gamma(S, A)] \implies [x \in \Gamma(T, B)]$,

(ii) $\Gamma(S, A) \cap T = \Gamma(T, A)$.

(iii) $\Gamma(\{y\}, A) = \{y\}$

The above says that (i) if an alternative x is considered in (S, A) then it will also be considered in (T, B) where T is a subset of S and B is a subset of A. In other words, if some alternative was considered in a set of alternatives and attributes, then it should be considered when the alternatives and the attributes set shrink, that is in such circumstances the DM is more focused, (ii) when the attributes set is fixed, the alternatives considered in T (the smaller set) are precisely those considered in its superset S ($T \subseteq S$), provided that the originally considered elements in S are still available in T, and (iii) when the feasible set of alternatives is a singleton, then that alternative is considered no matter what the set of attributes corresponds to it.

Although ACF is a general concept, it can be shown to incorporate the behaviour of a consideration set formed by a process (interpreted in an individual choice with attributes framework) known as "Voting by Committees" (see Barberà et al. (1991)). The Voting-by-Committees (VC) competition filter is similar in spirit to the latter applied to a competition filter. More explicitly:

$$\Gamma^{vc}(S, A) = \begin{cases} \{x \in S : v(x, a) = 1, \forall a \in A\}, & \text{if } |S| \geq 2; \\ S, & \text{if } |S| = 1. \end{cases}$$

where $S \in \mathcal{X}$, $A \in \mathcal{A}$, and $v : \mathbf{X} \times \mathbf{A} \to \{0, 1\}$ such that $v(x, a) = 1$ means that alternative x has the attribute a and $v(x, a) = 0$ means that alternative x does not have the attribute a. So, in menus with two or more alternatives, an alternative is considered if and only if it has all the relevant attributes. In menus with a single alternative, the alternative is always considered, which is a non-triviality assumption. It is easy to check that a VC competition filter is an ACF.

Next, we make a distinction with another paper Lleras et al. (2017) which is a prominent paper in this setting. Let Γ^L denote the competition filter used in the latter work which has the following property when modified to the setting with attributes:

if $x \in S \subseteq T$ and $x \in \Gamma^L(T)$ then $x \in \Gamma^L(S)$ for any $S \subseteq T \in \mathcal{X}$.[3]

Although, ACF looks similar to Γ^L, the two are logically distinct as our attributes set can vary across contexts. Formally, let $x \in T \subseteq S \in \mathcal{X}$ and $B \subseteq A \in \mathcal{A}$. If $x \in \Gamma^L(S, B)$ then $x \in \Gamma^L(T, A)$ but in ACF, $x \in \Gamma(S, B)$ does not necessarily imply $x \in \Gamma(T, A)$. Also, $x \in \Gamma^L(T, A)$ does not imply $x \in \Gamma^L(S, B)$. However, by ACF, $x \in \Gamma(T, A)$ implies $x \in \Gamma(S, B)$ since $[x \in \Gamma(T, A)] \implies [x \in \Gamma(T, B)] \implies [x \in \Gamma(S, B) \cap T] \implies [x \in \Gamma(S, B)]$ as $\Gamma(S, B) \cap T \subseteq \Gamma(S, B)$.

Let $c : \mathcal{X} \times \mathcal{A} \to \mathbf{X}$ be a *choice function* such that $c(S, A) \in S, \forall S \in \mathcal{X}, \forall A \in \mathcal{A}$. Let $\succeq$ be a *linear order* over $\mathbf{X}$ and we denote the best (or maximum) element in S with respect to $\succeq$ by $max(\succeq, S)$. We now define our choice rule.

Definition 2 *A choice function c is an Overwhelming Choice with Attributes **(OCA)** if there exists a linear order $\succeq$ and an ACF Γ such that $c(S, A) = max(\succeq, \Gamma(S, A))$.*

Therefore, a choice function is OCA if it can be rationalized by a preference relation $\succeq$ on X. We provide some examples.

Example 1 Suppose $\mathbf{X} = \{x, y, z\}$, $\mathbf{A} = \{a_1, a_2\}$. Suppose the choice function is as follows:

$$
\begin{aligned}
&c(\{x, y\}, \{a_1\}) = \{x\}, c(\{y, z\}, \{a_1\}) = \{y\}, c(\{x, z\}, \{a_1\}) = \{z\},\\
&c(\{x, y\}, \{a_2\}) = \{x\}, c(\{y, z\}, \{a_2\}) = \{y\}, c(\{x, z\}, \{a_2\}) = \{z\},\\
&c(\mathbf{X}, \{a_1\}) = \{y\}, c(\mathbf{X}, \{a_2\}) = c(\mathbf{X}, \mathbf{A}) = \{x\},\\
&c(\{x, y\}, \mathbf{A}) = \{x\}, c(\{y, z\}, \mathbf{A}) = \{y\}, c(\{x, z\}, \mathbf{A}) = \{z\},\\
&c(\{x\}, \{a_i\}) = \{x\}, c(\{y\}, \{a_i\}) = \{y\}, c(\{z\}, \{a_i\}) = \{z\} \text{ for } i \in \{1, 2\}.
\end{aligned}
$$

We will argue later that this choice function is not rationalizable since it does not satisfy the contraction consistency over fixed attributes (CCFA) defined in the next section. The axiom requires that if the relevant set of attributes does not change, then a contraction of the menu of alternatives (given that the choice in the original menu is still available) should not lead to choice changes. However, one may note that when the set of alternatives is $\mathbf{X}$ then x is chosen when the set of attributes is the full set $\mathbf{A}$, while y is chosen when the set of attributes is $\{a_1\}$. However, x is again chosen when the set of attributes is $\{a_2\}$. We will show later that is not ruled out by our main theorem. The reason for inconsistent choice under different attribute set could be due to different attention paid to different attributes. For example, it could be that under attribute a_2, alternative x ranks higher than y and under the presence of both attributes, a_2 receives more attention leading to the choice of x. When the

[3]Note that the competition filters in Lleras et al. (2017) is a mapping $\Gamma : \mathcal{X} \to \mathcal{X}$ since they do not model decision-making in the presence of attributes.

only attribute is a_1, alternative y be may ranked higher than x leading to the former being chosen.[4] This example shows that not all choice data are consistent with our axioms.

3 Axioms

We now introduce the axioms needed to characterize OCA.

Axiom 1 *Single Reversal in Attributes **(SRA)**: A choice function $c : \mathcal{X} \times \mathcal{A} \to \mathbf{X}$ satisfies **SRA** if for all $S \in \mathcal{X}$ and $A, B, D \in \mathcal{A}$ such that $D \subseteq B \subseteq A$ and $x, y \in S$,*

$$[x = c(S, A) \text{ and } y = c(S, B)] \implies [x \neq c(S, D)].$$

This axiom states that given a feasible set of alternatives S, if x is chosen when the relevant set of attributes is A and y is chosen when the relevant set of attributes is B such that $B \subseteq A$, then as the set of attributes shrink further to D such that $D \subseteq B$, x cannot be chosen from (S, D). This is because when A was the relevant set of attributes, it might have been the case that y was not considered. Since x was chosen (S, A), x is revealed to be considered in (S, A). As the attributes set shrinks, given that S is fixed, the DM is more focused. Hence, x cannot go from considered to not being considered. Also, since $y = c(S, B)$, y is revealed to be considered in (S, B), and since y was chosen when x was considered, y is revealed to be preferred to x. Now, since, $D \subseteq B \subseteq A$, x and y are both considered in D. This axioms prevents x being chosen in presence of y when both x and y are revealed to be considered.

Axiom 2 *Contraction Consistency with Fixed Attributes **(CCFA)**: A choice function $c : \mathcal{X} \times \mathcal{A} \to \mathbf{X}$ satisfies CCFA if for all $T, S \in \mathcal{X}$ such that $T \subseteq S \in \mathcal{X}$ and $x \in T \subseteq S$ and for all $A \in \mathcal{A}$,*

$$[x = c(S, A)] \implies [x = c(T, A)].$$

This is a variation of the standard contraction consistency in deterministic choice of alternatives (see Sen (1971)). This is a rationality postulate when the attribute sets are not varied across contexts which states that an alternative should continue to be chosen from a subset of the initial set of alternatives provided that it is still available.

We now show that **SRA** and **CCFA** are independent. Let $x, y \in T \subseteq S \in \mathcal{X}$ and $D \subseteq B \subseteq A \in \mathcal{A}$. Consider the following choice function: $x = c(S, A)$, $y = c(S, B)$,

[4] See Basu Mallik et al. (2024) for a model of stochastic choice with attributes and limited attention.

$x = c(S, D)$, $x = c(T, A)$, $y = c(T, B)$, and $x = c(T, D)$ where $x, y \in T \subseteq S \in \mathcal{X}$ and $D \subseteq B \subseteq A \in \mathcal{A}$. Thus, above choice data violates **SRA** since there are two choice reversals but satisfies **CCFA**.
Let $x = c(S, A)$, $y = c(S, B)$, $y = c(S, D)$, $y = c(T, A)$, $x = c(T, B)$, and $x = c(T, D)$. The above choice data is consistent with **SRA** but violates **CCFA**. Thus, **SRA** and **CCFA** are independent.

4 Main Result

We are ready to present our main result:

Theorem 1 *A choice function $c : \mathcal{X} \times \mathcal{A} \to \mathbf{X}$ is an **OCA** if and only if it satisfies **SRA** and **CCFA**.*

Proof. ($\Longleftarrow$) Suppose $c : \mathcal{X} \times \mathcal{A} \to \mathbf{X}$ is a satisfy axiom **SRA** and axiom **CCFA**. Take any $x, y \in \mathbf{X}$ and $A \in \mathcal{A}$. Define a binary relation $\succeq$ over $\mathbf{X}$ as follows:

$$[x \succeq y] \iff \Big[\textbf{Either} \quad \exists A, B \in \mathcal{A} \text{ such that } A \subseteq B \text{ and }$$
$$x = c(\{x, y\}, A) \text{ and } y = c(\{x, y\}, B) \textbf{ Or } x = c(\{x, y\}, H), \forall H \in \mathcal{A} \Big].$$

We show that $\succeq$ is a linear order. It is immediate that $\succeq$ is *reflexive*, as c is single-valued. We have

$$[x = c(\{x\}, A), \forall A \in \mathcal{A}] \implies [x \succeq x].$$

For completeness of $\succeq$, take any $x, y \in \mathbf{X}$. Consider $\mathbf{A}$ (the grand set of attributes) and the set $\{x, y\}$. As c is single-valued, assume without loss of generality, $c(\{x, y\}, \mathbf{A}) = x$. Then,

$$\textbf{Either } [\forall B \in \mathcal{A}, c(\{x, y\}, B) = x]$$
$$\textbf{Or } [\exists B \in \mathcal{A} \text{ such that } B \subseteq \mathbf{A} \text{ and } c(\{x, y\}, B) = y].$$

The former implies $x \succeq y$ and the latter implies $y \succeq x$. Thus, $\succeq$ is *complete*.

For antisymmetry, suppose $x \succeq y$ and $y \succeq x$. Then,

$$[x \succeq y] \implies \Big[\textbf{Either } [\exists A, B \in \mathcal{A}, A \subseteq B, c(\{x, y\}, A) = x, \text{ and } c(\{x, y\}, B) = y]$$
$$\textbf{Or } [c(\{x, y\}, F) = x, \forall F \in \mathcal{A}] \Big].$$
$$[y \succeq x] \implies \Big[\textbf{Either } [\exists C, D \in \mathcal{A}, C \subseteq D, c(\{x, y\}, C) = x, \text{ and } c(\{x, y\}, D) = y]$$
$$\textbf{Or } [c(\{x, y\}, G) = y, \forall G \in \mathcal{A}] \Big].$$

If either $c(\{x,y\},F) = x, \forall F \in \mathcal{A}$ or $c(\{x,y\},G) = y, \forall G \in \mathcal{A}$, then $x = y$ (as c is single-valued), which proves our claim. So, suppose otherwise. Then there exist $A, B, C, D \in \mathcal{A}$ such that $c(\{x,y\},A) = x$, $c(\{x,y\},B) = y$, $c(\{x,y\},C) = y$, and $c(\{x,y\},D) = x$, such that $A \subseteq B$ and $C \subseteq D$. Now, consider $A \cap C$. Note that $A \cap C \subseteq A \subseteq B$, $A \cap C \subseteq C \subseteq D$, and $A \cap C \in \mathcal{A}$. By **SRA**,

$$[c(\{x,y\},B) = y \text{ and } c(\{x,y\},A) = x] \implies [c(\{x,y\},A \cap C) = x].$$

Again by **SRA**,

$$[c(\{x,y\},D) = x \text{ and } c(\{x,y\},C) = y] \implies [c(\{x,y\},A \cap C) = y].$$

The above implies $x = y$. Thus, $\succeq$ is *antisymmetric*. We now show that $\succeq$ is transitive. Suppose for contradiction that $\succeq$ is not transitive. Then there exist $x, y, z \in \mathbf{X}$ such that $x \succeq y$, $y \succeq z$, and $x \not\succeq z$. By completeness of $\succeq$, $x \not\succeq z \implies z \succeq x$. Then,

$$[x \succeq y] \implies \Big[\textbf{Either } [\exists A, B \in \mathcal{A}, A \subseteq B, c(\{x,y\},A) = x, \text{ and } c(\{x,y\},B) = y] \\ \textbf{Or } [c(\{x,y\},K) = x, \forall K \in \mathcal{A}]\Big]$$

and,

$$[y \succeq z] \implies \Big[\textbf{Either } [\exists C, D \in \mathcal{A}, C \subseteq D, c(\{y,z\},C) = y, \text{ and } c(\{y,z\},D) = z] \\ \textbf{Or } [c(\{y,z\},L) = y, \forall L \in \mathcal{A}]\Big].$$

and,

$$[z \succeq x] \implies \Big[\textbf{Either } [\exists E, F \in \mathcal{A}, E \subseteq F, c(\{x,z\},E) = z, \text{ and } c(\{x,z\},F) = x] \\ \textbf{Or } [c(\{x,z\},M) = z, \forall M \in \mathcal{A}]\Big].$$

There are several cases to consider. Suppose, $c(\{x,y\},A) = x$, $c(\{x,y\},B) = y$, $c(\{y,z\},C) = y$, $c(\{y,z\},D) = z$, $c(\{x,z\},E) = z$, and $c(\{x,z\},F) = x$ such that $A \subseteq B$, $C \subseteq D$, and $E \subseteq F$. Take $G = A \cap C \cap E$. Note that $G \subseteq A \subseteq B$, $G \subseteq C \subseteq D$, and $G \subseteq E \subseteq F$. Then by **SRA**,

$$x = c(\{x,y\},G), y = c(\{y,z\},G), \text{ and } z = c(\{x,z\},G).$$

By the above and by **CCFA** (as G is fixed and $\{x,y\} \subseteq \{x,y,z\}$),

$$[y \neq c(\{x,y\},G)] \implies [y \neq c(\{x,y,z\},G)].$$

By **CCFA** again,

$$[z \neq c(\{y,z\},G)] \implies [z \neq c(\{x,y,z\},G)].$$

By the fact that c is single-valued, this implies that $c(\{x,y,z\},G) = x$. However, $z = c(\{x,z\},G)$, implies $x \neq c(\{x,z\},G)$.

By **CCFA**,

$$[x \neq c(\{x,z\},G)] \implies [x \neq c(\{x,y,z\},G)].$$

This is a contradiction.

Now let $x = c(\{x,y\},K), \forall K \in \mathcal{A}$, $y = c(\{y,z\},L), \forall L \in \mathcal{A}$, and $z = c(\{x,z\},M), \forall M \in \mathcal{A}$. Take any $G \in \mathcal{A}$. Then,

$$x = c(\{x,y\},G), y = c(\{y,z\},G), \text{ and } z = c(\{x,z\},G).$$

By applying **CCFA** as earlier, we get the desired contradiction.
Now let, $x = c(\{x,y\},K), \forall K \in \mathcal{A}$, $c(\{y,z\},C) = y$, $c(\{y,z\},D) = z$, $c(\{x,z\},E) = z$, and $c(\{x,z\},F) = x$ such that $C \subseteq D$, and $E \subseteq F$. Take $G = K \cap C \cap E$. Then, by **SRA**,

$$x = c(\{x,y\},G), y = c(\{y,z\},G), \text{ and } z = c(\{x,z\},G).$$

By applying **CCFA**, as earlier, we get our desired contradiction. By making similar arguments as the one made above, we can show that each of the remaining cases will also lead to a contradiction. Hence, $\succeq$ is *transitive*. Thus, $\succeq$ is a *linear order*.

Now define the mapping $\Gamma^m : \mathcal{X} \times \mathcal{A} \to \mathcal{X}$ recursively as:

$$(1)\ \Gamma^m(\mathbf{X},\mathbf{A}) = c(\mathbf{X},\mathbf{A})$$

$$(2)\ \Gamma^m(\mathbf{X},A) = \{y \in \mathbf{X} : y = c(\mathbf{X},B) \text{ for some } B \supseteq A\}$$

$$(3)\ \Gamma^m(S,A) = \{y \in S : y = c(S,B) \text{ for some } B \supseteq A$$

$$\text{and } y \in \Gamma^m(V,A) \text{ for all } V \supseteq S\}$$

$$(4)\ c(S,A) \in \Gamma^m(V,A), \forall V \supseteq S$$

$$(5)\ \Gamma^m(\{x\},A) = c(\{x\},A)$$

where $x \in \mathbf{X}$, $A, B \in \mathcal{A}$ and $S, V \in \mathcal{X}$ such that $|S| \geq 2$.

We show that Γ^m is an **ACF**[5]. First observe, by construction of the recursive definition, that $c(S,A) \in \Gamma^m(S,A)$ because $S \supseteq S$ and $c(S,A) \in \Gamma^m(V,A), \forall V \supseteq S$. Let $x \in T \subseteq S \in \mathcal{X}$ and $D \subseteq A \in \mathcal{A}$. By construction,

$$[x \in \Gamma^m(S,A)] \implies [\exists B \supseteq A \text{ such that } x = c(S,B)]$$

$$\implies [\exists B \supseteq A \supseteq D \text{ such that } x = c(S,B)].$$

By **CCFA**, $[x = c(S,B)] \implies [x = c(T,B)]$. Thus,

$$[x \in \Gamma^m(S,A)] \implies [\exists B \supseteq D \text{ and } x = c(T,B)].$$

To show that $x \in \Gamma^m(T,D)$ we must show that $x \in \Gamma^m(P,D), \forall P \supseteq T$, such that $P \in \mathcal{X}$.[6] But we already have that $x \in \Gamma^m(V,A), \forall V \supseteq S$ such that $V \in \mathcal{X}$ because $x \in \Gamma^m(S,A)$. So we just have to show that $x \in \Gamma^m(Z,A)$ $\forall Z \in \mathcal{X}$ such that $S \supseteq Z \supseteq T$.

As $x \in \Gamma^m(V,A), \forall V \supseteq S$ such that $V \in \mathcal{X}$, we have $x \in \Gamma^m(\mathbf{X},A)$. By the definition of Γ^m, $\exists M \supseteq A$ such that $x = c(\mathbf{X},M)$. By **CCFA**, we have $\forall Z \in \mathcal{X}$ such that $\mathbf{X} \supseteq S \supseteq Z \supseteq T$, $x = c(Z,M)$. Thus, $\exists M \supseteq A \supseteq D$ such that $x = c(Z,M)$. This implies that $x \in \Gamma^m(Z,A)$.

Now, to show that $x \in \Gamma^m(Z,D)$, we only need to show that $x \in \Gamma^m(W,D), \forall W \in \mathcal{X}$ where $S \supseteq W \supseteq Z$. We just need to need to repeat the same arguments as above[7] and we have our desired result that $x \in \Gamma^m(P,D), \forall P \supseteq T$, such that $P \in \mathcal{X}$. Thus, $x \in \Gamma^m(T,D)$.

Now, we show that $\Gamma^m(T,A) = \Gamma^m(S,A) \cap T$, whenever $T \subseteq S \in \mathcal{X}$ and $A \in \mathcal{A}$. This would establish our claim that Γ^m is an **ACF**. Let $T \subseteq S \in \mathcal{X}$ and $A \in \mathcal{A}$. Suppose $x \in \Gamma^m(S,A) \cap T$. Then,

$$[x \in \Gamma^m(S,A) \cap T] \implies [\exists B \supseteq A \text{ such that } x = c(S,B) \text{ and } x \in T].$$

Since $x \in T \subseteq S \in \mathcal{X}$, by **CCFA**,

$$[x = c(S,B)] \implies [c(T,B) = x] \implies [\exists B \supseteq A \text{ such that } x = c(T,B)]$$

To show that $x \in \Gamma^m(T,A)$, we just need to establish that $x \in \Gamma^m(P,A), \forall P \supseteq T$, such that $P \in \mathcal{X}$. But this follows by the exact same arguments as used above.

[5]Observe that (5) establishes requirement (iii) of an **ACF** as c is always single-valued.

[6]The arguments which follow apply for $x = c(S,A)$ as well by the construction of the recursive definition.

[7]Note that this process terminates at some stage as $\mathbf{X}$ is finite.

Hence,

$$x \in \Gamma^m(T, A)$$

Thus, we have:

$$\Gamma^m(S, A) \cap T \subseteq \Gamma^m(T, A).$$

Now let $x \in \Gamma^m(T, A)$. Since $T \subseteq S$, by the definition of Γ^m, $x \in \Gamma^m(S, A)$ and $x \in T$. Therefore, $[x \in \Gamma^m(S, A) \cap T] \implies [\Gamma^m(T, A) \subseteq \Gamma^m(S, A) \cap T]$. Therefore, $\Gamma^m(T, A) = \Gamma^m(S, A) \cap T$. This establishes that Γ^m is an **ACF**.

We already know that $c(S, A) \in \Gamma^m(S, A)$. Now, let $x \neq c(S, A)$ and $x \in \Gamma^m(S, A)$. This implies that $x \in S$ and $\exists B \supseteq A$ such that $x = c(S, B)$.

By **CCFA** (as $\{x, c(S, A)\} \subseteq S$, keeping B fixed), $x = c(\{x, c(S, A)\}, B)$. By applying **CCFA** again with $\{x, c(S, A)\} \subseteq S$, keeping A fixed,

$$c(S, A) = c(\{x, c(S, A)\}, A).$$

Therefore,

$$\exists A, B \in \mathcal{A}, \text{ such that } A \subseteq B, c(\{x, c(S, A)\}, A) = c(S, A),$$

$$\text{and } c(\{x, c(S, A)\}, B) = x.$$

By the construction of $\succeq$ and due to the fact that $x \neq c(S, A)$, we have $c(S, A) \succ x$, which implies that $c(S, A) \succ x, \forall x \neq c(S, A)$ and $x \in \Gamma^m(S, A)$, where $\succ$ is the asymmetric (strict) part of $\succeq$. Hence, if c satisfies **SRA** and **CCFA**, then c can be represented as an **OCA**, with $c(S, A) = \max(\succeq, \Gamma^m(S, A))$ where $\succeq$ and Γ^m are the linear order on **X** and the **ACF** on $\mathcal{X} \times \mathcal{A}$ respectively. This concludes the proof of the *if part* of the theorem.

($\implies$) We now prove the converse. Let c be representable by **OCA**, such that $c(S, A) = \max(\succeq, \Gamma(S, A))$, where $S \in \mathcal{X}, A \in \mathcal{A}$, $\succeq$ is a linear order on **X** and Γ is an **ACF**. Let $x = c(S, A)$ and $x \in T \subseteq S$.

$$\begin{aligned}
&\implies x \succeq y, \forall y \in \Gamma(S, A) \\
&\implies x \succeq y, \forall y \in \Gamma(S, A) \cap T \subseteq \Gamma(S, A) \\
&\implies x \succeq y, \forall y \in \Gamma(T, A) = \Gamma(S, A) \cap T \\
&\implies x = \max(\succeq, \Gamma(T, A)) \\
&\implies x = c(T, A), \text{ with } T \subseteq S
\end{aligned}$$

Therefore, c satisfies **CCFA**.

For **SRA**, let $x = c(S, A)$, $y = c(S, B)$, and $D \subseteq B \subseteq A$. Then,

$$x = \max(\succeq, \Gamma(S, A)) \text{ and } y = \max(\succeq, \Gamma(S, B)).$$

We have, by definition of **OCA** and **ACF**,

$$\begin{aligned}
& x = \max(\succeq, \Gamma(S, A)) \\
& \implies [x \in \Gamma(S, A)] \\
& \implies [x \in \Gamma(S, B), \text{ as } B \subseteq A] \\
& \implies [y \in \Gamma(S, B), x \in \Gamma(S, B), \text{ and } y \succ x \text{ as } y = \max(\succeq, \Gamma(S, B))] \\
& \implies [y \in \Gamma(S, D), x \in \Gamma(S, D), \text{ and } y \succ x \text{ as } D \subseteq B] \\
& \implies [x \neq \max(\succeq, \Gamma(S, D))] \\
& \implies [x \neq c(S, D), \text{ whenever } D \subseteq B \subseteq A].
\end{aligned}$$

Thus, c satisfies **SRA**. Thus, the *Only If part* of theorem is now established. This concludes the proof of the theorem. ■

5 Endogenous Attributes Filter

In the previous section, we had considered the case where the attributes considered were exogenously visible i.e. all the attributes were assumed to be considered. Moreover, they were assumed to be driving limited attention over the set of alternative and there was a preference relation over the set of alternatives. In this section, we consider the dual problem: The alternatives considered are exogenously visible i.e. all the alternatives are considered and limited attention is paid to the attributes available while the preference relation is over the set of alternatives via individual attributes.

We posit that when more alternatives are present, they distract the DM. Hence, the DM can focus on less number of attributes and make a decision based on the preference between the alternatives as per these less number of considered attributes.

Formally we define the following:

Definition 3 *Endogenous Attribute Filter(**EAF**): Endogenous Attribute Filter is a mapping $\gamma : \mathcal{X} \times \mathcal{A} \to \mathcal{A}$ such that:*

$$\gamma(U, B) \subseteq B, \forall U \in \mathcal{X}, \forall B \in \mathcal{A}.$$

and

$$a \in \gamma(S, A) \implies a \in \gamma(T, A)$$

where $a \in A \in \mathcal{A}$ *and* $T \subseteq S \in \mathcal{X}$.

EAF simply states that if some attribute is considered when the choice problem is (S, A) then the same attribute is also considered when the menu of alternatives shrink to $T \subseteq S$, given that the relevant attributes set is fixed. It also states that if an attribute is not relevant (to the given choice problem) then it is not considered, in effect, only relevant attributes can be considered.

We now introduce a novel behavioural postulate:

Definition 4 *Contraction Consistent Attribute Removal Reversal (**CCARR**): A choice function c is said to satisfy Contraction Consistent Attribute Removal Reversal (**CCARR**) if* $c(S, A) \neq c(S, A \setminus \{a\})$ *then* $c(T, A) \neq c(T, A \setminus \{a\})$*, where* $a \in A \in \mathcal{A}$ *and* $T \subseteq S \in \mathcal{X}$.

This axiom states that if the removal of an attribute from a relevant attribute set A causes a change in choice when the menu of alternatives is S then the removal of the same attribute from the same relevant attribute set also causes a change in choice from a submenu $T \subseteq S$ of alternatives. We now show that **CCARR** is strictly weaker than **CCFA** given an additional availability restriction.

Proposition 1 ***CCFA** implies **CCARR** if the choices involved in the choice reversal of the superset are still avaiable in the subset but not conversely.*

Proof. Let c satisfy **CCFA** and let $c(S, A) \neq c(S, A \setminus \{a\})$. As entailed in the statement of the claim, let $c(S, A), c(S, A \setminus \{a\}) \in T$. By **CCFA**, $c(S, A) = c(T, A)$ and $c(S, A \setminus \{a\}) = c(T, A \setminus \{a\})$, where $c(S, A), c(S, A \setminus \{a\}) \in T \subseteq S$. Hence, $c(T, A) = c(S, A) \neq c(S, A \setminus \{a\}) = c(T, A \setminus \{a\})$. Therefore, $c(T, A) \neq c(T, A \setminus \{a\})$. Thus, c satisfies **CCARR**.

Now take $x = c'(S, A)$, $y = c'(S, A \setminus \{a\})$, $z = c'(T, A)$, and $w = c'(T, A \setminus \{a\})$ such that $x, y, z, w \in T \subseteq S$. Assuming x, y, z, and w are all distinct, we have $c'(S, A) \neq c'(T, A)$ and $c'(S, A \setminus \{a\}) \neq c'(T, A \setminus \{a\})$. Hence, c' does not satisfy **CCFA**. However, $c'(S, A) \neq c'(S, A \setminus \{a\})$ and $c'(T, A) \neq c'(T, A \setminus \{a\})$. Hence, c' satisfies **CCARR**. Thus, **CCARR** is strictly weaker than **CCFA**. ■

CCARR can be interpreted in our framework in the following intuitive fashion. If the removal of an attribute a from the relevant attribute set A can induce a change in choice in menu (of alternatives) S, then it can be inferred that the attribute grabs attention in such a choice problem (S, A). **CCARR** requires that the same attribute also grabs attention in a submenu of alternatives $T \subseteq S$ with the same set of relevant attributes attached to it. We can explicitly construct the **EAF** in such cases:

$$\gamma^m(S, A) = \{a \in A : c(S, A) \neq c(S, A \setminus \{a\})\}$$

where $S \in \mathcal{X}$ and $A \in \mathcal{A}$.

To see that that γ^m is indeed an **EAF**, let $T \subseteq S \in \mathcal{X}$ and $a \in A \in \mathcal{A}$. Take $a \in \gamma^m(S, A)$. Then $c(S, A) \neq c(S, A \setminus \{a\})$. By **CCARR**, $c(T, A) \neq c(T, A \setminus \{a\})$. This implies $a \in \gamma^m(T, A)$ which establishes our claim. Hence, **CCARR** is sufficient for constructing an **EAF**.

One can posit that the DM prefers to make a choice being more informed, in the sense that she likes considering more attributes while making her choice as her orderings on the alternatives are attribute based. This in turn means (given the definition of **EAF**) that the DM prefers making a choice from a smaller set of alternatives. The above is consistent with the analysis by Lleras et al. (2017). Hence, this hypothesis of preference for more attributes provides an attribute-based backing of their analysis. However, providing an explicit characterization of that is a non-trivial problem and we leave that for future work.

For now, we can use the revealed preference methodology to elicit attribute orderings as in Basu Mallik and Bhowmik (2024):

$$x \succeq_a y \iff x = c(\{x, y\}, \{a\})$$

where $x, y \in \mathbf{X}$ and $a \in \mathbf{A}$. It is straightforward to get a utility representation (see Basu Mallik and Bhowmik (2024) for more details):

$$U : \mathbf{X} \times \mathbf{A} \to \mathbf{R}_+$$

$$U(x, a) \geq U(y, a) \iff x \succeq_a y$$

We now define a choice rule based on *maximin preferences*, see Gilboa and Schmeidler (1989), in terms of attributes and limited attention:

Definition 5 *A choice rule c is said to satisfy Constrained Maximin (**CMM**) if:*

$$c(S, A) = argmax_{x \in S} min_{a \in \gamma(S,A)} U(x, a)$$

*where $S \in \mathcal{X}$, $A \in \mathcal{A}$, and γ is an **EAF**.*

To see that the above problem of characterizing preference for more attributes (information) (and hence less alternatives) is non-trivial we provide an example where there are 'Double Reversals': $\exists x, y \in V \subseteq T \subseteq S \in \mathcal{X}$ such that $x = c(V, A), y = c(T, A)$, and $x = c(S, A)$. On the brighter side of things **CMM** will be able to explain violations of the choice rule posited in Lleras et al. (2017). The above double reversals also violate Weak-WARP (see Manzini and Mariotti (2007)) when $V = \{x, y\}$.

Example 2 ***CMM** and **Double Reversals**:*

Let $U(x,a) = 6, U(x,b) = 7, U(x,c) = 2, U(x,d) = 8,$

$U(y,a) = 5, U(y,b) = 4, U(y,c) = 3, U(y,d) = 1.5,$

$U(z,a) = 2, U(z,b) = 1, U(z,c) = 4, U(z,d) = 0.5$

$U(w,a) = 3, U(w,b) = 2, U(w,c) = 1, U(w,d) = 0.25$

Let $x = c(\{x,y,z,w\},\{a,b,c,d\}), y = c(\{x,y,z\},\{a,b,c,d\})$ *and* $x = c(\{x,y\},\{a,b,c,d\})$. *Suppose* $\gamma(\{x,y,z,w\},\{a,b,c,d\}) = \{a,b\}$, $\gamma(\{x,y,z\},\{a,b,c,d\}) = \{a,b,c\}$, *and* $\gamma(\{x,y\},\{a,b,c,d\}) = \{a,b,c,d\}$. *Then* c *is consistent with* ***CMM*** *and also exhibits* ***Double Reversals****.*

In the above example, it is straightforward to check that γ is an **EAF** and c satisfies **CMM**. Note that c exhibits *Double Reversals* because $x = c(\{x,y,z,w\},\{a,b,c,d\}), y = c(\{x,y,z\},\{a,b,c,d\})$, $x = c(\{x,y\},\{a,b,c,d\})$, and $\{x,y\} \subseteq \{x,y,z\} \subseteq \{x,y,z,w\}$. The attributes set is fixed at $\{a,b,c,d\}$ and the choice is reversed from x (in $\{x,y,z,w\}$) to y (in $\{x,y,z\}$) and back to x (in $\{x,y\}$).

6 Conclusion

We consider a model of individual choice with menus and sets of attributes with competition filters. We provide necessary and sufficient conditions for rationalizability of choice functions through an attributes competition filter. The dual of this problem was presented in Section 5 where the attention on the set of attributes was limited. We hope that future work can provide further insights to the conditions under which a choice function can be rationalized in such settings. The use of double reversals in choice may be required along with additional axioms of revealed preference appropriate for the modified setting with attributes.

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