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Attention and framing*

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Abstract

We consider individual decision-making where every alternative appears with a *frame* (á la Salant and Rubinstein (2008)). The decision maker is subject to inattention due to framing effects that leads to random choice. We characterize a *frame-based stochastic choice rule* according to which the choice probability of an alternative (say, x) is the probability with which attention is drawn by its frame and not by the frames which are associated with the alternatives that beat x according to a complete binary relation.

JEL classification: D81, D90

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1 Introduction

In any decision-making situation, individual choice is often observed to be random. There are many papers which deal with random choice (e.g. Yildiz (2016), Manzini and Mariotti (2014), Manzini and Mariotti (2015), Block et al. (1959), Ahn et al. (2017)). In the literature, the randomness has been attributed to variation in attention paid by the decision maker to different alternatives (e.g. Manzini and Mariotti (2014), Block et al. (1959)). Limited attention has been explained primarily as an inability or cognitive constraint of the decision maker to consider all the items (or alternatives). However, these papers do not model the source of the inattention. In our paper, we model variability in attention by explicitly including the effects of various *frames* with which the alternatives appear.

In our model, the decision maker chooses from a set of alternatives, where every alternative appears with a *frame*. Our formulation of framing effect is similar to that of Salant and Rubinstein (2008). A "product" in our model is an alternative (x) with an associated frame (i). For instance, consider a chocolate packaged in a 'gift-box', where chocolate is an alternative and 'gift-box' is a frame. The randomness in the decision maker's attention could depend on the nature and quality of the frames. Certain types of packaging may be more attractive to the decision-maker. Thus, the probability with which a product draws the decision-maker's attention is a function of the frame. This is consistent with theory and experimental evidence.[1]

In this paper, we characterize a stochastic choice rule called *frame-based stochastic choice rule* which can be stated as follows: there exists a complete binary relation over the set of alternatives and an attention function that assigns a probability to every frame such that for any set of products G, the choice probability of x_i in G is the probability that attention is drawn by frame i and not by frames attached to those alternatives in G that beat x according to the binary relation. The binary relation and the attention function in the rule are identifiable. We interpret the attention parameter as the probability with which a particular frame draws the decision maker's attention. We illustrate the rule with an example:

Example: Suppose that there are two available shelves in a store: *top* and *bottom*. Each shelf (frame) can accommodate either of the following two brands of soft-drinks: a and b.

Suppose b is displayed in the *top* shelf and a is displayed in the *bottom* shelf. Therefore, $(b, top\ shelf)$ and $(a, bottom\ shelf)$ are two products. Let

[1] For a broad survey of this literature see Kahneman and Tversky (2000) and related works. For empirical evidence see (Sigman (2015)).

$G = \{(a, bottom\ shelf), (b, top\ shelf)\}$. Suppose that the binary relation $\succeq$ is such that $a \succeq b$ and δ is the attention parameter such that $\delta(top) = 0.8$ and $\delta(bottom) = 0.2$. The choice probabilities according to the frame-based stochastic choice rule are as follows:

(i) Probability of choosing a is the probability that the *bottom* shelf draws attention: $P(a, G) = \delta(bottom) = 0.2$.

(ii) Probability of choosing b is the product of the probability that attention is drawn by the *top* shelf and not by the *bottom* shelf as it contains a (the alternative that beats b): $P(b, G) = \delta(top) \times [1 - \delta(bottom)] = 0.8(1 - 0.2) = 0.64$.

(iii) Probability of not choosing anything (say x^*) is the residual probability: $P(x^*, G) = 1 - 0.2 - 0.64 = 0.16$.

Notice that even though a beats b, due to the lower attention drawn by the frame in which a appears (*bottom*), b is chosen with a higher probability than a.

We characterize frame-based stochastic choice rule using three axioms: *invariance of singletons*, *dominance* and *stochastic path independence.*

Invariance of singletons states that the probability of an alternative when no other alternative is present only depends on the frame attached to it. Dominance has two parts. Informally, part (i) of dominance states that, given a set of products G, if an alternative x dominates any other alternative y from G in a pairwise comparison, then its choice probability is the same as its choice probability when it appears alone, i.e., when the set of products is just $\{x_i\}$. Dominance (ii) states that for a given set of products G if x dominates y in a pairwise comparison, when they appear in frame i and j respectively, then x dominates y when they appear in any other set of frames k and l respectively as well.

The third axiom is similar to *stochastic path independence* in Yildiz (2016) and i-independence axiom in Manzini and Mariotti (2014).

The literature on random choice rules is rich (E.g. Manzini and Mariotti (2014), Ahn et al. (2017), Li and Tang (2017) and Fudenberg et al. (2015), Caplin and Martin (2018).) The paper closest to ours is Manzini and Mariotti (2014) which characterizes a stochastic choice rule with limited attention. However, our paper differs from theirs in a significant way. In their paper, the attention function only depends on the alternatives; in our model, the attention depends on the frames. Therefore, they do not model the source of the inattention.

Another paper that deals with effect of frames on stochastic choice is Caplin and Martin (2018). They model the welfare aspects of frame-based individual decision making. In their paper, a planner uses framing as an information structure to influence decision making. Caplin and Martin (2018) extends this idea to stochastic choice as well; however it does not explicitly model the randomness in attention attributed to framing.

Ahn et al. (2017) provides a characterization of Luce rule and Fudenberg et al. (2015) provides a cardinal treatment of random choice with a perturbed utility model. Li and Tang (2017) show that stochastic choice rules are backwards-induction rationalizable. However, none of these papers model the source of the inattention.

To the best of our knowledge this is the first paper to model framing effects as the source of randomness in attention. The rule we characterize clearly demonstrates the effects of frames in drawing a consumer's attention to a product. Our framework extends to settings where the frames are positions in a list (Rubinstein and Salant (2006)) or ordered trees (Mukherjee (2014)).

The paper is organized as follows. Section 2 describes the model and discusses the axioms. Section 3 provides the main result and some examples. The proofs are presented in the Appendix.

2 Model

Let X be the set of all alternatives and F be the set of all frames. The pair (x, i) is a product where alternative $x \in X$ appears with the frame $i \in F$. For simplicity we denote the product (x, i) by x_i.

We say that an alternative x and a frame i are *compatible* if x_i is a well-defined product. For example, if $x = soft\ drink$ and $i = paper\ bag$ then $(x, i) = (soft\ drink, paper\ bag)$ is not a well-defined product since *soft drink* cannot be packaged in a *paper bag*. All such combinations of alternatives and frames which are incompatible are excluded from our domain. We assume that every alternative in X is *compatible* with every frame in F. An implication of this is the following. If $(x, i) = (soft\ drink, tetra\ pack)$ and $(y, j) = (milk, glass\ bottle)$ are well defined products, then the products $(y, i) = (milk, tetra\ pack)$ and $(x, j) = (soft\ drink, glass\ bottle)$ are also well-defined.

Let $\bar{A} : X \times F$ be the set of all such well-defined products. The decision maker has an option not to choose any product in which case we assume that a default product x^* is chosen (e.g. exiting the browser window on an online shop without buying anything). Denote $A : \bar{A} \cup \{x^*\}$.[2]

[2]It follows that every product in our domain consists of an alternative with a *compatible*

For any $G \subseteq \bar{A}$, let $G^* = G \cup \{x^*\}$ be the set of products containing the default alternative, x^*, and let $G(X) \subseteq X$ and $G(F) \subseteq F$ be the set of alternatives and the set of frames attached to the alternatives in G, respectively.

A *stochastic choice rule* is a mapping $P : A \times 2^{\bar{A}} \to [0,1]$ such that $\sum_{x_i \in G^*} P(x_i, G) = 1$. Note that the probability with which the default alternative is chosen is $P(x^*, G) = 1 - \sum_{x_i \in G; x_i \neq x^*} P(x_i, G)$ for any $G \subseteq A$.

Therefore, for any set of products $G \subseteq A$, $P(x_i, G)$ is the probability of choosing the alternative $x \in X$ with frame $i \in F$ from the given set of products G. When the set of products G is empty the decision maker chooses the default product x^*. Therefore, $P(x^*, \phi) = 1$. We define the following class of stochastic choice rules.

Definition 1 (Frame-based stochastic choice rule) A stochastic choice rule $P : A \times 2^{\bar{A}} \to [0,1]$ is a *frame-based stochastic choice rule* if there exists a function $\delta : F \to (0,1)$ and a complete binary relation[3] $\succeq$ over X such that for any $x \in X$, $i \in F$ and $G \subseteq A$ where $x_i \in G$,

$$P(x_i, G) = \delta_i \prod_{j \in G(F) | y \succeq x, y_j \in G \setminus \{x_i\}} (1 - \delta_j)$$

where we denote $\delta(i)$ and $\delta(j)$ by δ_i and δ_j respectively for brevity. Here, δ_i, which we call the attention parameter or the attention probability, is the probability with which frame i draws the decision maker's attention. Note that it is independent of the alternative associated with the given frame.

According to the above rule, the choice probability of x_i is the probability that the frame i draws the decision maker's attention and the frames associated with the alternatives that beat x according to $\succeq$ do not draw attention. In the Appendix we provide conditions under which $\succeq$ is transitive.

We compare the above rule to the random consideration set (rcs) rule in Manzini and Mariotti (2014):

Definition 2 (Random consideration set rule (rcs) (MM (2014)) A random consideration set rule is a random choice rule $p_{\succ,\gamma}$ for which there exists a pair $(\succ, \gamma)$, where $\succ$ is a strict total order on X and γ is a map $\gamma : X \to (0,1)$, such that,

$$p_{\succ,\gamma}(a, A) = \gamma(a) \sum_{b \in A : b \succ a} (1 - \gamma(b)) \text{ for all } A \in D \text{ and } a \in A.$$

where D is the relevant domain containing subsets of X. In the rcs rule, randomness arises due to a two-stage decision making process in which the

frame.

[3]A binary relation $\succeq$ is complete over X if $\forall x, y \in X$, $x \succeq y$ or $y \succeq x$.

decision maker considers only those alternatives that attract attention. The above rule is a choice rule from a set of alternatives. Therefore, there is no concept of frames in their model. Consider the following example:

Example 3 Let $G_1 = \{x_i, y_j, z_k\}$ and $G_2 = \{x_j, y_k, z_i\}$. The set of alternatives in G_1 and G_2 are the same: $\{x, y, z\}$. Let $y \succ x$, $x \succ z$ and $y \succ z$. According to the rcs-rule, x will be chosen with the same probability $\gamma(x)(1-\gamma(y))$ from both G_1 and G_2. However, according to the frame-based stochastic choice rule, the probability of choosing x_i from G_1 may differ from the probability of choosing x_j from G_2 since $P(x_i, G_1) = \delta_i(1-\delta_j)$ is not necessarily equal to $\delta_j(1-\delta_k) = P(x_j, G_2)$.

In Manzini and Mariotti (2014), the binary relation is strict. However, we can define a weak order version of the rcs-rule as follows: $p_{\succeq,\gamma}(a, A) = \gamma(a)\prod_{b\in A\setminus\{a\}:b\succeq a}(1-\gamma(b))$ for all $A \in D$ and $a \in A$. Suppose $y \succeq x$, $x \succeq z$ and $y \succeq z$. Again, x will be chosen with the same probability from both G_1 and G_2 according to the weaker version of the rcs-rule, but the choice probabilities of x_i may differ according to the frame-based stochastic choice rule.

Further, the attention probability and the binary relation are both defined over the set of alternatives in the rcs-rule while the attention probabilities are independent of the alternatives in the frame-based stochastic choice rule. Therefore, unlike the frame based stochastic choice rule, the role of frames in attracting the decision maker's attention cannot be explained by the rcs-rule.

The following axioms characterize the frame-based stochastic choice rule.

2.1 Axioms

Invariance of singletons (IS) For all $x, y \in X$ and $i \in F$, $P(x_i, \{x_i\}) = P(y_i, \{y_i\})$.

IS requires that the choice probabilities of any two products from singleton sets are equal when the two products have the same frame. This axiom emphasizes that choice is stochastic due to the frame attached to an alternative. Notice that this axiom is an independence requirement on the choice probability from singleton sets.

Dominance (DOM) Consider any $x \in X$ and $i \in F$. We have the following:

(i) for any $G \subseteq \bar{A}$ such that $x_i \in G$, suppose that $P(x_i, \{x_i, y_j\}) > P(y_i, \{y_i, x_j\})$ $\forall\ y \in X$ and for some $j \in F$, such that $y_j \in G$. Then $P(x_i, G) = P(x_i, \{x_i\})$;

(ii) suppose that $P(x_i, \{x_i, y_j\}) > P(y_i, \{y_i, x_j\})$ for some $y \in X$ and for some $j \in F$. Then, $P(x_k, \{x_k, y_l\}) > P(y_k, \{y_k, x_l\})$ for all $k, l \in F$.

We say that x *dominates* y when x_i is chosen with a higher probability from $\{x_i, y_j\}$ than y_i from $\{x_j, y_i\}$, for any $i, j \in F$. Note that this is antecedent of DOM (i) and (ii).[4]

DOM (i) requires that if for some $x_i \in G \subseteq \bar{A}$, x 'dominates' y in the above sense for all y such that $y_j \in G$, then the choice probability of x_i from G is the same as the probability with which x_i is chosen from the singleton $\{x_i\}$. This axiom is an independence requirement similar in spirit to the classical independence of irrelevant alternatives axiom in the context of frames: if an alternative x 'dominates' all the other alternatives in a given set, its choice probability from that set is independent of the presence of the dominated alternatives.

DOM (ii) requires that if an alternative x dominates y when x is assigned frame i and y is assigned frame j, then x dominates y when x and y are assigned any two frames $k, l \in F$. This axiom rules out situations in which x dominates y for a pair of frames $\{i, j\}$, and y dominates x for another pair of frames $\{k, l\}$.

Stochastic Path Independence (SPI) For any $x, y \in X$ and $i, j \in F$ such that $x_i, y_j \in G$ for any $G \subseteq \bar{A}$, if $P(x_i, \{x_i, y_j\}) \leq P(y_i, \{y_i, x_j\})$, then,

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = 1 - P(y_j, \{y_j\}).$$

We introduce the notion of 'impact' in order to explain the above axiom. Suppose y dominates x in the manner described above. For a given set of products G, following Manzini and Mariotti (2014), we define the *impact* of removing y_j from G on x_i as the ratio $\frac{P(x_i, G \setminus \{y_j\})}{P(x_i, G)}$. SPI states that if y dominates x, then the impact of removing y_j from G on x_i is the probability of not choosing y_j from the singleton set $\{y_j\}$. This axiom is the stochastic version of the path-independence condition of Plott (1973) and similar to the axiom of the same name in Yildiz (2016) and Manzini and Mariotti (2014).

3 The Result

Theorem 4 A stochastic choice rule P is a *frame-based stochastic choice rule* if and only if it satisfies *IS*, *DOM* and *SPI*.

[4] Note that DOM (ii) implies that the 'dominance' relation is unique for any distinct $x, y \in X$.

Theorem 4 characterizes the class of frame-based stochastic choice rules. For any two alternatives, the binary relation shows which alternative beats the other. According to the rule, randomness in choice data arises due to the attention probabilities of the frames with which the alternatives appear.

The formal proof is provided in the Appendix. Here, we provide a sketch: using 'dominance' we define a weak binary relation $\succeq$ over the set of alternatives. DOM (ii) ensures that $\succeq$ is complete. We derive the choice probability of x_i from a set of products G as follows: we partition G into subsets such that x beats all the alternatives in one subset, and all the alternatives beat x in the other subset. SPI allows us to express $P(x_i, G)$ as $P(x_i, G \setminus \{y_j\})(1 - P(y_j, \{y_j\})$ for each $y \succeq x$. We iteratively remove all such y_j from G. Notice that x beats all the alternatives that remain in the set of products (except itself) according to $\succeq$. DOM(i) renders these products irrelevant. The attention parameter is identified as the probability with which a product is chosen from a set that contains no other products. IS ensures that the attention parameter does not depend on the alternatives, but is simply a function of the frame.

There are several environments where the frames and the attention probabilities have a nice structure. We provide some examples in the next subsection.

3.1 Frames and attention parameter: some examples

The set of frames often exhibits interesting structures. For instance, packaging quality for an item can vary from a basic quality to premium - here frames can be ordered on a continuum. We give examples of some environments where frames can be ordered and the attention probabilities for different frames depend on the order.

(i) Ordered frame-based stochastic choice rules: suppose that the frames are ordered according to popularity or quality. Let this ordering be denoted by $<$, where $i < j$ implies that frame i is less popular than frame j. It is natural to assume that $\delta_i \leq \delta_j$ if $i < j$, i.e. a more popular frame is likely to draw attention with higher probability. According to the frame-based stochastic choice rule, when an alternative is in a frame that is more popular than the other frames it will have a higher probability of being chosen by the decision maker, given the binary relation.

(ii) Primacy and recency: alternatives appear in a sequence of frames where $i < j$ implies that frame i is observed before frame j; this is the positioning of the frames. For example, a menu in a restaurant or items placed on a shelf in a shop. There can be many possible relationships between this sequence and the attention parameter. For example, if $i < j \Rightarrow \delta_i \geq \delta_j$, then this signifies 'primacy effect' where an alternative has a higher chance of being chosen by the decision maker when it appears at the beginning of the list or menu (Rubinstein and Salant (2006)).

Alternatively, if $i < j \Rightarrow \delta_i \leq \delta_j$, this is the 'recency effect' (Rubinstein and Salant (2006)) where the decision maker chooses an alternative with higher probability when it appears at the end of the list.

The frame-based stochastic choice rule recognizes that the frame attached to an alternative may influence the probability with which it is chosen. The stochastic parameter and the binary relation over the alternatives is identified[5].

4 Conclusion

We model choice behavior in the setting of imperfect attention. In our model, the source of imperfect attention are the frames attached to the alternatives. We characterize frame-based stochastic choice rule, in which the frames draw the decision maker's attention irrespective of the alternatives in the frame. The attention probabilities and the underlying binary relation are identifiable.

5 Appendix

5.1 Proofs of theorems

Proof of Theorem 4. Necessity: Let P be a frame based stochastic choice rule and $\succeq$ be the associated complete binary relation over X.

IS: By definition $\delta_i \in (0,1)$ for all $i \in F$. Therefore, $P(x_i, \{x_i\}) = \delta_i = P(y_i, \{y_i\})$, $\forall\ x, y \in X$. Hence, IS is necessary.

DOM(i): Suppose that $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$, $\forall\ y_j \in G \setminus \{x_i\}$, $G \subseteq \bar{A}$. There are three possible cases:

[5]See proof of Theorem 4.

(i) $x \succ y : P(x_i, \{x_i, y_j\}) = \delta_i$ and $P(y_i, \{x_j, y_i\}) = \delta_i(1 - \delta_j)$;

(ii) $y \succ x : P(x_i, \{x_i, y_j\}) = \delta_i(1 - \delta_j)$ and $P(y_i, \{x_j, y_i\}) = \delta_i$;

(iii) $x \sim y : P(x_i, \{x_i, y_j\}) = \delta_i(1 - \delta_j)$ and $P(y_i, \{x_j, y_i\}) = \delta_i(1 - \delta_j)$.

Since, $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$, only (i) is feasible (since $0 < \delta_i, \delta_j < 1$). Thus $x \succ y$, $\forall\, y \in G(X)$. It follows from the rule that $P(x_i, G) = \delta_i = P(x_i, \{x_i\})$.

DOM(ii): Suppose that $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$ for some $y \in X, j \in F$. Following the same argument as above (proof of necessity of DOM(i)), we can show that $x \succ y$. Thus for any $k, l \in F$, $P(x_k, \{x_k, y_l\}) = \delta_k$ and $P(y_k, \{x_l, y_k\}) = \delta_k(1 - \delta_l)$. Hence $P(x_k, \{x_k, y_l\}) > P(y_k, \{x_l, y_k\})$, since $0 < \delta_k, \delta_l < 1$.

SPI: Take any $x, y \in X$, and $i, j \in F$ such that $x_i, y_j \in G$ for some $G \subseteq \bar{A}$. Again, either $x \succ y$ or $y \succeq x$. By similar arguments as above, $P(x_i, \{x_i, y_j\}) \leq P(y_i, \{x_j, y_i\})$ only if $y \succeq x$. Then,

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = \frac{\delta_i \prod_{w_r \in G \setminus \{x_i\} | w \succeq x}(1 - \delta_r)}{\delta_i \prod_{w_r \in G \setminus \{x_i\}, w \neq y | w \succeq x}(1 - \delta_r)} = 1 - \delta_j = 1 - P(y_j, \{y_j\}).$$

If $x \succ y$, then as argued previously, the antecedent of SPI will not hold. Therefore, it is satisfied vacuously. We now prove sufficiency.

Sufficiency: Suppose that P satisfies IS, DOM and SPI.

We define the binary relation $\succeq$ over X as follows: for any $x, y \in X$, $x \succeq y \iff P(x_i, \{x_i, y_j\}) \geq P(y_i, \{y_i, x_j\})$ for all $i, j \in F$. Moreover,

(i) $x \succ y \iff (x \succeq y)$ and $\neg(y \succeq x)$;

(ii) $x \sim y \iff (x \succeq y)$ and $(y \succeq x)$.

We show that $\succeq$ is complete. Without loss of generality (WLOG), we have the following two cases for any $x, y \in X$:

(i) Suppose $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$ for some $i, j \in F$. By DOM(ii), $P(x_k, \{x_k, y_l\}) > P(y_k, \{x_l, y_k\})$ $\forall\, k, l \in F$. By definition of $\succeq$, $x \succ y$.

(ii) Suppose $P(x_i, \{x_i, y_j\}) = P(y_i, \{x_j, y_i\})$ for some $i, j \in F$. We can show that $P(x_k, \{x_k, y_l\}) = P(y_k, \{y_k, x_l\})$ for all $k, l \in F$ as follows: suppose $P(x_k, \{x_k, y_l\}) \neq P(y_k, \{y_k, x_l\})$ for some $k, l \in F$,

$k \neq i$ or $l \neq j$. WLOG let $P(x_k, \{x_k, y_l\}) > P(y_k, \{x_l, y_k\})$. By DOM (ii), $P(x_m, \{x_m, y_n\}) > P(y_m, \{x_n, y_m\}) \; \forall \; m, n \in F$. This implies that $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$. This is a contradiction. Therefore, if $P(x_i, \{x_i, y_j\}) = P(y_i, \{x_j, y_i\})$ for some $i, j \in F$, then $P(x_k, \{x_k, y_l\}) = P(y_k, \{x_l, y_k\})$ for all $k, l \in F$. By definition, $x \sim y$. Therefore $\succeq$ is complete. Reflexivity of $\succeq$ follows from part (ii) above for any $x, y \in X$ such that $x = y$.

Now, consider any $x_i \in G, G \subseteq \bar{A}$. We partition G as follows: $G = G_1 \cup G_2 \cup \{x_i\}$ where $G_1 = \{w_r \in G \setminus \{x_i\} | P(x_i, \{x_i, w_r\}) \leq P(w_i, \{x_r, w_i\})\}$ and $G_2 = \{w_r \in G \setminus \{x_i,\} | P(x_i, \{x_i, w_r\}) > P(w_i, \{x_r, w_i\})\}$.

By definition of $\succeq$ and DOM (ii):

$$w \succeq x, \; \forall \; w_r \in G_1 \text{ and } x \succ w, \; \forall \; w_r \in G_2.$$

Pick an arbitrary $y_j \in G_1$. By SPI,

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = 1 - P(y_j, \{y_j\}).$$

This in turn implies that

$$P(x_i, G) = P(x_i, G \setminus \{y_j\})(1 - P(y_j, \{y_j\})). \tag{1}$$

We pick another arbitrary product $q_l \in G_1 \setminus \{y_j\}$. By a similar argument as the one used above for y_j,

$$P(x_i, G \setminus \{y_j\}) = (1 - P(q_l, \{q_l\}))P(x_i, G \setminus \{y_j, q_l\}) \tag{2}$$

The equations 1 and 2 imply,

$$P(x_i, G) = (1 - P(y_j, \{y_j\}))(1 - P(q_l, \{q_l\}))P(x_i, G \setminus \{y_j, q_l\}).$$

By the repeated application of the above steps for every $y_j \in G_1$,

$$P(x_i, G) = P(x_i, G \setminus G_1) \prod_{y_j \in G_1} (1 - P(y_j, \{y_j\}))$$

,

$$P(x_i, G) = P(x_i, G_2 \cup \{x_i\}) \prod_{y_j \in G_1} (1 - P(y_j, \{y_j\})). \tag{3}$$

Now consider $G_2 \cup \{x_i\}$. By construction of G_2, for all $w_r \in G_2$, $P(x_i, \{x_i, w_r\}\}) > P(w_i, \{w_i, x_r\})$.

Therefore, by DOM(i),

$$P(x_i, G_2 \cup \{x_i\}) = P(x_i, \{x_i\}). \quad (4)$$

From (3) and (4) we get: $P(x_i, G) = P(x_i, \{x_i\}) \prod_{y_j \in G_1}(1 - P(y_j, \{y_j\}))$. By IS we know that for any $i \in F$, $P(x_i, \{x_i\}) = P(y_i, \{y_i\})$ for all $x, y \in X$. Define $\delta : F \to (0, 1)$ as follows:

$$P(x_i, \{x_i\}) = \delta_i \text{ for any } i \in F \text{ and for any } x \in X.$$

Therefore, $P(x_i, \{x_i\}) = P(y_i, \{y_i\}) = \delta_i$ for $i \in F$, for any $x, y \in X$.

Using the definition of the binary relation $\succeq$ and by the construction of G_1, $P(x_i, G) = P(x_i, \{x_i\}) \prod_{y_j \in G \setminus \{x_i\} | y \succeq x}(1 - P(y_j, \{y_j\}))$.

Using the definition of δ in $P(x_i, G)$, we get

$$P(x_i, G) = \delta_i \prod_{j \in G(F) | y \succeq x, y_j \in G \setminus \{x_i\}} (1 - \delta_j).$$

Note that the proof remains unchanged when either G_1 or G_2 or both are empty[6].

□

5.2 Independence of axioms

Axioms characterizing the frame-based stochastic choice rule: We show that the axioms IS, DOM and SPI are independent of each other:

- IS: Define $\alpha : X \to (0.5, 1)$. $\succeq$ is a complete binary relation over X such that for all $x_i \in G, G \subseteq \bar{A}$,

  $$P(x_i, G) = \alpha(x) \prod_{y \in G(X): y \succeq x; y_j \neq x_i}(1 - \alpha(y)).$$

  The above rule is similar to the stochastic choice rule characterized in Manzini and Mariotti (2014). It satisfies DOM (i), DOM (ii) and SPI but does not satisfy IS.

[6] If $G_1 = \phi$ then $G = G_2 \cup \{x_i\}$ and $P(x_i, G) = P(x_i, G_2 \cup \{x_i\})$. Using DOM (i) and setting $P(x_i, \{x_i\}) = \delta_i$, we get $P(x_i, G) = P(x_i, \{x_i\}) = \delta_i$. If G_2 is empty, then $P(x_i, G) = P(x_i, G_1 \cup \{x_i\})$. By the repeated application of SPI for all $y_j \in G_1$, we get $P(x_i, G) = P(x_i, \{x_i\}) \prod_{y_j \in G_1}(1 - P(y_j, \{y_j\}))$. Setting $P(x_i, \{x_i\}) = \delta_i$, $P(y_j, \{y_j\}) = \delta_j$ as above and using the definition of $\succeq$, $P(x_i, G) = \delta_i \prod_{j \in G(F) | y \succeq x; y_j \in G \setminus \{x_i\}}(1 - \delta_j)$.

- DOM (i): Let $\delta : F \to (0.5, 1)$ and $\succeq$ be a complete binary relation over X. For all $x_i \in G$, $G \subseteq \bar{A}$ consider the following stochastic choice rule:

$$P(x_i, G) = \delta_i \prod_{j \in G(F) | y_j \in G \setminus \{x_i\}, x \succeq y} \delta_j \prod_{k \in G(F) | y_k \in G \setminus \{x_i\}, y \succeq x} (1 - \delta_k)$$

  The above rule satisfies IS, SPI and DOM (ii) but does not satisfy DOM (i).

- DOM (ii): Let $\delta : F \to (0, 1)$ and $\succeq$ be a complete binary relation over X. Let $D \subset A$ such that for any $z_k, m_r \in D$, $z \neq m$ and $k \neq r$. For all $x_i \in G$, $G \subseteq D$ and some $i^* \in F$ consider the following stochastic choice rule:

$$P(x_i, G) = \begin{cases} \delta_i \prod_{j \in G(F): x \succeq y, y_j \in G \setminus \{x_i\}} (1 - \delta_j) & \text{when } i = i^* \\ \delta_i \prod_{j \in G(F): y \succeq x, y_j \in G \setminus \{x_i\}} (1 - \delta_j) & \text{otherwise} \end{cases}$$

  This rule satisfies IS, DOM (i) and SPI but does not satisfy DOM (ii).

- Let $\delta : F \to (0, 1)$ and $\succeq$ be a complete binary relation over X. For all $x_i \in G$, $G \subseteq \bar{A}$ consider the following stochastic choice rule:

$$P(x_i, G) = \delta_i \prod_{j \in G(F): y_j \in G \setminus \{x_i\}, y \succeq x} \delta_j$$

  The above rule satisfies IS and DOM (i) and DOM (ii) but does not satisfy SPI.

5.3 The binary relation and transitivity

In our model, the binary relation $\succeq$ defined over X is not required to be transitive. Imposing the following axiom makes it transitive.

Transitivity (TR): For any $x, y, z \in X$ and for any $i, j \in F$, if $P(x_i, \{x_i, y_j\}) \geq P(y_i, \{x_j, y_i\})$ and $P(y_i, \{y_i, z_j\}) \geq P(z_i, \{y_j, z_i\})$ then

$$P(x_i, \{x_i, z_j\}) \geq P(z_i, \{x_j, z_i\}).$$

TR rules out cycles in the choice probabilities in binary sets when the frames are interchanged.

Definition 5 *A stochastic choice rule* $P : A \times 2^{\bar{A}} \to [0,1]$ *is a* P^* *rule if there exists a function* $\delta : F \to (0,1)$ *and a complete, transitive binary relation* $\succeq$ *over* X *such that for any* $x \in X$*, any* $i \in F$ *and* $G \subseteq \bar{A}$ *where* $x_i \in G$*,*

$$P(x_i, G) = \delta_i \prod_{j \in G(F): y \succeq x, y_j \in G \setminus \{x_i\}} (1 - \delta_j)$$

Corollary 6 *A stochastic choice rule* P *is a* P^* *rule if and only if it satisfies IS, DOM, SPI and TR.*

Proof. We show the necessity of TR. The rest of the proof is similar to the proof of theorem 4.

Necessity: Let P be a P^* rule and $\succeq$ be a complete, transitive binary relation over X. Let $x \succeq y$ for some $x, y \in X$. By the definition of the rule, $P(x_i, \{x_i, y_j\}) = \delta_i$ and $P(y_i, \{x_j, y_i\}) = \delta_i(1 - \delta_j)$. Since $\delta \in (0,1)$, $P(x_i, \{x_i, y_j\}) \geq P(y_i, \{x_j, y_i\})$. Suppose $y \succeq z$ for some $z \in X$.Since $\succeq$ is transitive, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$. Given $\delta \in (0,1)$, $P(x_i, \{x_i, z_j\}) = \delta_i \geq \delta_i(1 - \delta_j) = P(z_i, \{z_i, x_j\})$. Therefore TR is necessary. ■

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