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Communicating Bias

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Communicating Bias*

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Abstract

We consider a static cheap talk model in an environment with either one or two experts whose biases are privately known by the experts themselves. Before the experts learn the state, they send a cheap talk message about their bias to the decision maker. Subsequently, the decision maker chooses one expert to get state relevant advice from. We ask two questions - One, is there an equilibrium where the experts' bias is fully revealed? Two, is the bias revealing equilibrium welfare improving for the decision maker? We find that when there is only one expert, there is no bias revealing equilibrium. However, if there are two experts, there exists a bias revealing equilibrium, and under some conditions it gives the decision maker more utility than any equilibrium which is possible without bias revelation. This highlights a new channel through which sender competition can benefit the decision maker.

JEL codes: D82, D83

Keywords - Cheap talk, uncertain bias, multiple senders, bias revelation

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1 Introduction

Consider an individual who wants to consult a doctor. He realizes that the doctor's preference may not be perfectly aligned with her own. In particular, a very conservative doctor may under-prescribe medicines, while there may be other types of doctors who will want her to spend more money than required on his treatment. Suppose he is uncertain about how conservative the doctor is. Therefore, when the doctor recommends spending a certain amount of money for her treatment, he is unsure about both a) the bias of the doctor and b) the true state of her illness. In many other real-life situations, a decision maker has to decide which expert to colnsult, where the experts have privately known biases. For medical advice, one has to choose among several doctors, a defendant in a legal case has to hire one from a pool of lawyers, for financial advice one has to hire one of several financial managers etc.

Motivated by such examples, we consider a cheap talk game with $N \in \{1,2\}$ senders, where the bias of each sender is her private knowledge. It is common knowledge that the bias can be either *high* or *low* and that the bias of all senders is chosen independently from a known distribution. Before any player gets information about the payoff relevant state, we add a pre-play bias communication stage (stage 1). In this stage, the senders simultaneously send cheap talk messages about their bias type, following which the receiver selects one sender. Once selected, the sender can perfectly observe the true payoff relevant state, which is commonly known to have a uniform distribution. The next stage (stage 2) is a standard cheap talk game where the chosen sender observes the true state perfectly and sends a cheap talk message about it to the decision maker. The decision maker chooses an action and all players get paid.

We ask two questions. One, is there an equilibrium where the sender(s) reveals their bias truthfully, and what role does sender competition play? Two, if such an equilibrium exists, does it give the decision maker higher utility than any equilibrium where the senders' bias is not revealed in the pre-play bias communication? While exogenous bias revelation in a cheap talk game has been studied (Li and Madarász (2008)), to the best of our knowledge, ours is the first to study endogenous bias revelation by the senders, and explore the conditions required for all types of senders to reveal their bias, and the impact of these conditions on equilibria and welfare.

We start with a baseline case of one sender only and then analyze the case of two senders. We find that there does not exist an equilibrium where the sender truthfully reveals her bias when there is only one sender. The reason for this is as follows. First, note that any bias revealing equilibrium will feature a Crawford-Sobel type partition equilibrium (Crawford and Sobel (1982)) in the second stage. Suppose that there is a bias revealing equilibrium where the high bias message results in a m partition equilibrium and the low bias message results in a n partition equilibrium. If $m \leq n$, it is easy to show that the high bias sender would deviate and pretend to be low bias since she will be able to obtain higher actions in equilibrium, whereas the low bias sender would want to announce her type honestly. That is, the gain from deviation[1] is non-negative for the high bias sender and the gains from not deviating is non-negative for the low bias sender. Furthermore, we show that the former is higher than the latter for any $m \geq n$. Thus, as we increase m to incentivize the high bias sender to not deviate, before the gains from deviating becomes negative for the high bias sender, the gains from not deviating becomes negative for the low bias sender. Therefore, we cannot incentivize both types of senders to reveal their bias truthfully in any equilibrium.

However, when two senders compete to be hired by the decision maker, there exists a bias revealing equilibrium. The two sender case presents two new forces. First, since the decision maker hires only one expert, the senders are competing to get hired. Second, since the probability of selection depends on every sender's bias message, there is an element of strategic uncertainty that is not present in the one sender case. These new forces allow us to control incentives by identifying conditions on the outside option and on the strategic uncertainty which allow the incentive compatibility constraints to hold.

Next, we want to understand the welfare implication of competition and, in particular, we would like to know when the bias revealing equilibrium is welfare improving for the decision maker. When the model permits a public randomization device, we find an example where the receiver prefers a bias revealing equilibrium over any equilibrium that exists without the bias revelation stage. Since bias revelation is only possible with multiple senders, we conclude that competition can improve welfare in this game. The fact that sender competition can influence decision maker welfare by influencing the incentive of the senders to reveal their biases demonstrates a new channel through which competition can impact the welfare of the decision maker.

[1] Payoff from deviation minus the payoff from announcing own type correctly.

We contribute to the literature on cheap talk games with uncertain sender bias. The paper closest to ours is Li and Madarász (2008) which discusses a static cheap talk game with one sender of unknown bias. They compare two regimes - one where the sender must announce her true bias before communicating about the state, and another where the sender has no possibility of revealing any information about her bias before sending state relevant information to the decision maker. They find that if the utility function of the decision maker is concave enough, then he may prefer the regime where the sender's bias is not revealed.

Our paper differs from Li and Madarász (2008) in two ways. One, we allow the senders to choose if they want to reveal their biases by adding a pre-play bias communication stage before any expert gets to observe the true state. This is quite different than the exogenous bias revelation regime considered in Li and Madarász (2008): firstly, since a sender's bias is her private information, in most environments, it would be very difficult to justify how an exogenous truthful bias revelation can be enforced. This assumption becomes harder to justify because we show that in the one sender case endogenous bias revelation is, in fact, not possible. Furthermore, while Li and Madarász (2008) considers equilibria that follow exogenous true bias announcement, they do not consider the conditions needed for the sender to endogenously reveal her bias prior to learning the state. These conditions limit the kind of outcomes possible in equilibrium. The second point of difference from Li and Madarász (2008) is that we allow for more than one sender. We show that while endogenous bias revelation is not possible with one sender, bias revealing equilibria exist in the two sender world, and they can even be welfare improving for the decision maker.

Quement (2016) also considers a model with unknown bias. In his paper, there are two senders and the receiver gets messages from both the senders sequentially. One important difference between this setup and ours is that, in our model, the receiver can only get message from one sender and moreover, the sender has the option to reveal biases. Thus, in contrast to Quement (2016), where an increase in the number of senders reduces the receiver's payoff, we find competition can improve the welfare of the receiver.

The second strand of literature we connect to is the cheap talk models with multiple senders. Li (2010) considers a model with multiple senders and privately known bias with three different protocols of learning, namely, sequential, simultaneous, and hierarchical. This paper finds that competition is welfare improving and simultaneous learning to be the most efficient of all three. In contrast, we introduce sender competition in a different way: only one sender is hired after the pre-play bias communication stage, so the senders compete to get hired, without knowing the state. We state conditions for which this can be welfare improving. So, we explore the role of competition in bias revelation as well. Li et al. (2016) also considers a model of cheap talk with multiple senders, where each sender gets a private signal about their own project. In contrast, in our model, there is only one payoff relevant state; and once hired, a sender learns perfectly about that state.

The rest of the paper is organized as follows: section 2 describes the model. In section 3, we start with a baseline case of one sender and show that there does not exist a bias revealing equilibrium in the one sender model (subsection 3.1). Subsequently, in section 3.2 we show that such an equilibrium does exist in the two-sender model. Next, we show that a bias-revealing equilibrium may give the decision maker more utility than any equilibrium possible without bias revelation.

2 Model

Primitives

We consider a one-shot strategic communication game with one decision maker (he) and $N \in \{1,2\}$ experts (she) $(S_1,...,S_n)$. The state of the world θ is commonly known to be uniformly distributed on the unit interval $[0,1]$. The decision maker can hire exactly one of the experts to get state-relevant advice from. If an expert is hired, she learns the state perfectly and can send a cheap talk message (m) to the decision maker. Following this message, the decision maker takes an action $y(m)$. All experts are biased in that their preferences do not align perfectly with the decision maker's preferences. An expert S_i's bias b_i is her private information, but it is common knowledge that biases are drawn IID from the distribution:

$$b_i = \begin{cases} b_h & with\ probability\ p_h \in (0,1) \\ b_l & with\ probability\ 1 - p_h \end{cases}$$

where $|b_l| < |b_h|$. We assume that biases are low enough ($|b_i| \leq \frac{1}{4}\ \forall\ i$) so that there exists at least one informative cheap talk equilibrium when the hired expert's bias is known.

Since biases are unknown, we consider a two-stage game. In the first stage, the players play a cheap talk game which could reveal information about the type of the expert, i.e., her bias.[2] Following this, in the second stage, the hired expert and the decision maker play a standard cheap talk game where the expert sends a message to the DM about the payoff relevant state.

To contrast our results with those of the nondisclosure world in Li and Madarász (2008), we will often consider a strategic communication game where the experts do not have the option to reveal their types in the first stage. This game will not have the first stage game. We will call this the *LM* game. The analysis for such a game would follow the non-disclosure environment analysis presented in Li and Madarász (2008).

Timing

The timing of the game is as follows. At the beginning of the game in stage 1, each expert S_i privately learns her own bias $b_i \in \{b_h, b_l\}$ and then simultaneously sends a costless message $m_b^i \in \mathscr{M}_b$ to the DM that potentially conveys information about their own bias. Without loss of generality, we focus on direct mechanisms, so, $\mathscr{M}_b = \{b_h, b_l\}$. The decision maker then chooses to hire one expert as her advisor according to a hiring rule $h : \mathscr{M}_b \longrightarrow \Delta\{S_1, ...S_N\}$, where the hiring rule depends on the observed message vector sent by the N experts, and $h_i(m_b)$ denotes the probability of hiring expert S_i if the message vector is $m_b = (m_b^1, ...m_b^N)$. Note that, if $N = 1$, the DM hires the expert for sure.

In stage 2, the hired expert i learns the true state θ perfectly and sends another message $m_\theta^i \in \mathscr{M}$ to the DM possibly conveying some information about the state. Focusing on direct mechanisms, We assume $\mathscr{M} = [0, 1]$. Upon observing m_θ^i, the DM takes an action $y\left(m_\theta^i\right) \in [0, 1]$. Note that stage 2 looks like Crawford and Sobel (1982) if the bias of the chosen expert is fully revealed in stage 1. If not, then stage 2 looks like the non-disclosure world of Li and Madarász (2008). The expert who is not hired gets her outside option (described under 'Payoffs' below).

Beliefs

Let $P_b^i \in [0, 1]$ denote the posterior belief that the DM forms about the hired expert i being high bias (b_h) type at the end of stage 1 upon receiving bias-relevant information from all the experts. Let

[2]Note that an expert learns the state only after being hired. Therefore, in stage 1, the only information that can be conveyed is about the experts' bias.

$P \in \Delta(\theta)$ be the posterior belief about true state θ that the DM forms in stage 2 upon receiving state-relevant information from the hired expert.

Payoffs

If the true state is θ, the hired expert is expert i, and the decision maker takes the action y, then the payoffs are as follows:

$$\begin{aligned} U_{DM}(\theta,y) &= -(y-\theta)^2 \\ U_i(\theta,y,b_i) &= -(y-\theta-b_i)^2 \\ U_{j\neq i} &= -A_{b_j} \end{aligned}$$

where the expert who is not selected (for $N=2$) gets a reservation payoff of $-A_b$ for $b \in \{b_l, b_h\}$. We assume that $A_l = A, A_h = A+c$, so $c \geq 0$ captures the idea that the high bias sender's outside option is allowed to be worse than the low bias sender's outside option. To make sure that experts always want to get hired, we assume their reservation payoff is weakly worse than the lowest possible equilibrium payoff obtained by any expert from being hired, that is - worse than the payoff of from a babbling equilibrium in the Crawford-Sobel world.

Strategies

We will consider only pure strategies for the experts. A pure strategy for an expert S_i with bias b_i consists of two functions. The first function $\mu_{bi} : \{b_l, b_h\} \to \mathscr{M}_b$ determines the bias message sent by the expert in stage 1. This depends upon the expert's own true bias. The second function $\mu_{\theta i} : \{b_l, b_h\} \times P_b^i \times \theta \to \mathscr{M} = [0,1]$ determines the message ($m_\theta^i \in [0,1]$) about the state $\theta \in [0,1]$ sent by the expert i in stage 2 if hired. This message depends upon the chosen expert's true bias in $\{b_l, b_h\}$, the decision maker's posterior belief (P_b^i) that the chosen expert is type b_h, and the true state θ observed by the expert.

The DM's strategy also consists of two functions. One, a hiring function $h : \mathscr{M}_b{}^N \to \Delta\{S_1, ...S_N\}$ which determines which expert to hire as a function of the stage 1 vector of messages. Two, an action function $y : P_b^i \times \mathscr{M} \to [0,1]$ which determines the action taken as a function of the belief about the chosen expert and the message sent by the chosen expert in stage 2.

Perfect Bayesian Equilibrium

A Perfect Bayesian equilibrium consists of a profile of strategies for the decision maker and all experts, and belief vectors P, P_b such that: given the strategies of all players, the beliefs are derived using Bayes' rule whenever possible. The decision maker's hiring rule h and the action function y, maximize his ex-ante expected utility given his belief P, P_b and the strategy of all experts. We can write the DM's problem as,

$$\max_{y,h} EU_{DM}(P(P_b(m_b), m_\theta^i), y(P_b(m_b), m_\theta^{h(m_b)}))$$

For each type of expert, their strategy should maximize their expected payoff given the strategies of all the other experts and the decision maker. We can write expert i's problem as

$$\max_{m_b^i, m_\theta^i} EU_i(P(P_b(m_b), m_\theta^i), y(P_b(m_b), m_\theta^{h(m_b)}), b_i)$$

Notation

For ease of exposition, let us denote $CS_{b_{j'}}^j(k)$ as the payoff for a b_j type sender in a k partition equilibrium when the sender's bias is thought to be $b_{j'}$ with probability 1. Thus, when senders are truth-telling, from Crawford and Sobel (1982), we get:

$$CS_{b_h}^h(m) = -\frac{1}{12m^2} - \frac{b_h^2(m^2+2)}{3}$$
$$CS_{b_l}^l(n) = -\frac{1}{12n^2} - \frac{b_l^2(n^2+2)}{3}$$

In the next section, we will find expressions for the payoffs $CS_{b_l}^h(n)$ and $CS_{b_h}^l(m)$ and how they impact equilibrium. An n partition cheap talk equilibrium between a sender of known type b and the receiver will be denoted as 'n partition CS b equilibrium' (since this will be exactly the same as a n partition equilibrium in Crawford and Sobel (1982)).

3 Analysis

We first study our benchmark case of one sender. We explore if truthful bias revelation is possible in equilibrium in stage 1. We show that this is not possible in the one sender world. However, we show in later sections that truthful bias revelation is possible in the two sender world. Moreover, under some conditions, the bias revealing equilibrium can give the decision maker a higher utility than any feasible bias hiding equilibrium. This analysis demonstrates a new channel through which competition and strategic uncertainty amongst senders can improve the welfare of the decision maker. In the two following subsections we show the role of competition and strategic uncertainty on the incentive compatibility constraints of the different types.

3.1 One sender world

Is it possible that the experts reveal their biases perfectly in stage 1 in some equilibria? We start our analysis by exploring the possibility of such bias revealing equilibria existing when there is only one sender. This will serve as our baseline case. Before going further, we note that it is obvious that if the receiver plays a babbling equilibrium in stage 2 irrespective of the messages received in stage 1, then both types of senders will have no incentive to deviate from truth telling in the bias revealing stage. However, since this equilibrium is uninteresting[3], we will henceforth only consider those equilibria in stage 2 where the receiver is playing a non babbling cheap talk equilibrium after at least one of the stage 1 messages.

Our main result in this subsection is that with one sender, there does not exist a bias revealing equilibrium in pure strategies. This is not intuitive at first glance. In stage 2 of the game, the receiver can promise different cheap talk equilibria as a reward to the sender for revealing her bias in stage 1. For example, the DM may compensate the higher bias sender with a finer partition equilibria[4] for revealing her bias as compared to the equilibrium following the lower bias revelation.[5] However, we show that no matter which equilibria are offered for revealing their true bias types in stage 2,

[3]Though the bias is revealed in equilibrium, this is not payoff relevant for the decision maker.

[4]To the extent allowed by the size of the bias.

[5]The reader may wonder if the lower bias sender will want to deviate and lie about her type to benefit from the finer partition offered to the higher bias sender in such an equilibrium. The lower bias sender's incentive compatibility constraint will be satisfied if the partition points of the high bias sender are unbalanced (many small partitions at the lower levels of the state with larger partitions at the higher levels) enough. This is because of the risk averse utility function of the senders.

either the low bias sender or the high bias sender (or both) will want to deviate from a bias revealing equilibrium in stage 1. This is in contrast to the two sender case (section 3.2) where we show that there exist equilibria where the senders truthfully reveal their bias in equilibrium. Proposition 1 shows our main result for this section.

Proposition 1. *When there is only one sender* ($n = 1$)*, there is no bias revealing equilibrium in pure strategies.*

Proof. We will prove this result by contradiction. Suppose there exists a bias revealing equilibrium in pure strategies. WLOG, let the equilibrium be the following:

Stage 1:

$\mu_b(x) = x \; \forall \; x \in \{b_l, b_h\}$

$h(b) = 1 \; \forall \; b \in \{b_l, b_h\}$

Stage 2:

If sender reports type b_l in stage 1: Play an n partition CS b_l equilibrium

If sender reports type b_h in stage 1: Play an m partition CS b_h equilibrium

If the decision maker arrives at an off equilibrium node, she takes the lowest equilibrium action in n partition CS b_l equilibrium

First, let us consider the incentives of the high bias sender. If she plays according to the strategies proposed above, her expected payoff is:

$$\frac{-1}{12m^2} - \frac{{b_h}^2(m^2+2)}{3} \tag{1}$$

Clearly there is no reason to deviate in stage 2 of the game if she reveals her type truthfully in stage 1 (since stage 2 play is an equilibrium, there is no incentive to deviate). If she deviates in stage 1 and reports her type to be b_l, then in stage 2 she can exploit the n partition CS b_l equilibrium to her advantage. In particular, while she does not have the incentives to deviate from the equilibrium messages (else the decision maker plays the action $\frac{1}{2}$), she will change the interval of the state space on which the messages are reported (a la Li and Madarasz's conflict hiding equilibrium). In an n partition CS b_l equilibrium, the equilibrium actions are given by

$$y_i = \frac{2i-1}{2n} + b_l(2i^2 + (1+n)(1-2i))$$

where $i = 1,2,...,n$. Now, in equilibrium, the high bias expert will not deviate from the messages the low bias expert was meant to send in equilibrium (else the dm takes the action half). However, the high bias expert does not have to choose the same partition function as the low bias expert. In fact, she will choose cut off points on the state space to maximize her own payoff from the equilibrium messages. In particular, in equilibrium, she will choose points $a_1,...,a_{n-1}$ such that $a_i + b_h = \frac{y_i + y_{i+1}}{2}$. When the state is between a_i and a_{i+1}, the sender will send the message so that action y_i will be played in response. The expected payoff to the high bias expert from deviating is therefore given by:

$$CS^h_{b_l}(n) = \int_0^{a_1} -(y_1 - \theta - b_h)^2 \, d\theta + \int_{a_1}^{a_2} -(y_2 - \theta - b_h)^2 \, d\theta + \ldots \int_{a_{n-1}}^{1} -(y_n - \theta - b_h)^2 \, d\theta$$

Substituting the expressions for y_i and a_i and simplifying, we get that the expected payoff to the high bias expert from deviating is

$$CS^h_{b_l}(n) = \frac{-1}{12n^2} + {b_l}^2\left(\frac{4}{3} - \frac{1}{n} - \frac{n^2}{3}\right) + b_l b_h \frac{2(1-n)}{n} - \frac{{b_h}^2}{n} \tag{2}$$

Comparing 1 and 2, we get that the high bias sender will not deviate if:

$${b_l}^2\left(4 - \frac{3}{n} - n^2\right) + {b_h}^2\left(m^2 + 2 - \frac{3}{n}\right) - b_l b_h \frac{6(n-1)}{n} \;\leq\; \frac{1}{4}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{3}$$

Inequality 3 captures the incentive compatibility constraint of the high bias expert for the prescribed strategies to constitute an equilibrium.

Now, let us consider the incentives of the low bias sender. Doing the same analysis as before, we can show that the low bias expert will not deviate from the prescribed strategies if:

$$-{b_h}^2\left(4 - \frac{3}{m} - m^2\right) - {b_l}^2\left(n^2 + 2 - \frac{3}{m}\right) + b_l b_h \frac{6(m-1)}{m} \;\geq\; \frac{1}{4}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{4}$$

Looking at the bias revealing incentives of the two types of senders jointly, we see that 3 and 4 can simultaneously hold only if:

$$-{b_h}^2(4-\frac{3}{m}-m^2)-{b_l}^2(n^2+2-\frac{3}{m})+b_l b_h\frac{6(m-1)}{m} \geq {b_l}^2(4-\frac{3}{n}-n^2)+{b_h}^2(m^2+2-\frac{3}{n})-b_l b_h\frac{6(n-1)}{n} \tag{5}$$

$$\iff (b_h-b_l)^2(2-\frac{1}{n}-\frac{1}{m}) \leq 0 \tag{6}$$

This inequality cannot hold unless $b_h = b_l$ or $n = m = 1$ (only babbling equilibrium is played). Since we have assumed that $b_l < b_h$ and we are only looking for non trivial equilibria in stage two, we conclude that the proposed strategies do not constitute an equilibrium since at least one type of sender will have incentives to deviate from truth telling in period 1. □

The intuition behind the non existence of a bias revealing equilibrium in the one sender case is as follows. Suppose that there is a bias revealing equilibrium where the high bias message results in a m partition equilibrium and the low bias message results in a n partition equilibrium. If $m \leq n$, it is easy to show that the high bias sender would deviate and pretend to be low bias since she will be able to obtain higher actions in equilibrium, whereas the low bias sender would want to announce her type honestly. That is, the gain from deviation is non-negative (i.e. $CS^h_{b_l}(n) - CS^h_{b_h}(n) \geq 0$) for the high bias sender and the gains from not deviating is non-negative (i.e. $CS^l_{b_l}(n) - CS^l_{b_h}(n) \geq 0$) for the low bias sender. Now consider any $m > n$ and the difference between these two differences i.e.:

$$\begin{aligned}&[CS^h_{b_l}(n) - CS^h_{b_h}(m)] - [CS^l_{b_l}(n) - CS^l_{b_h}(m)] \\ &= (b_h-b_l)^2(2-\frac{1}{n}-\frac{1}{m}) + \frac{(b_h^2-b_l^2)}{3}(m^2-n^2)\end{aligned} \tag{7}$$

This expression is clearly positive since $m > n$, $b_h > b_l$ and at least one of m, n is greater than 1 (else it would be a babbling equilibrium and we have stated before - we are interested in only informative equilibria). Thus, as we increase m to incentivize the high bias sender to not deviate, before the gains from deviating becomes negative for the high bias sender, the gains from not deviating becomes negative for the low bias sender. Therefore, we cannot incentivize both types of senders to reveal their bias truthfully in any equilibrium.

So, what kind of equilibria exist when there is only one sender? From our result, we know that any equilibrium will be one where there is uncertainty about the sender's bias in stage 2. In these environments, 'conflict hiding'[6] equilibria may exist. One example is as follows:

Example 1. *Suppose* $b_l = \frac{1}{6}, b_h = \frac{1}{5}$ *and* $p_h = \frac{3}{5}$. *The following strategy profile constitutes a perfect Bayesian Equilibrium.*

Stage 1

$\mu_b(x) = b_l \ \forall \ x \in \{b_l, b_h\}$

$h(b) = 1 \ \forall b \in \{b_l, b_h\}$

Stage 2

Sender Strategy:

$\mu_{sb_l} = m_1 \ if \ \theta \in [0, 0.146]$

$\mu_{sb_l} = m_2 \ if \ \theta \in (0.146, 1]$

$\mu_{sb_h} = m_1 \ if \ \theta \in [0, 0.113]$

$\mu_{sb_h} = m_2 \ if \ \theta \in (0.113, 1]$

Decision maker strategy:

If DM observes b_l *in stage 1 and* m_1 *in stage* $2 = 0.0631$

If DM observes b_l *in stage 1 and* m_2 *in stage* $2 = 0.5631$

If DM observes b_l *in stage 1 and* $m \neq m_1, m_2$ *in stage* $2 = 0.0631$ *If DM observes* b_h *in stage* $1 = 0.5$

[6] See Li and Madarasz 2008

$Beliefs$

$P_b = p_h\ \forall$ *messages in stage* 1

$P = p(b = b_h/m_1)U[0,0.146] + p(b_l/m_1)U[0,0.113]$ *; if DM observes b_l in stage 1 and m_1 in stage 2*

$P = p(b_h/m_2)U[0.146,1] + p(b_l/m_2)U[0.113,1]$ *; if DM observes b_l in stage 1 and m_2 in stage 2*

$P = p(b_h/m_1)U[0,0.146] + p(b_l/m_1)U[0,0.113]$ *; if DM observes b_l in stage 1 and $m \neq m_1, m_2$ in stage 2*

$P = U[0,1]$ *; if DM observes b_h in stage 1* (8)

The decision maker's expected payoff in this equilibrium is -0.056.

3.2 Two sender world

Now, we consider the environment with two experts. After stage 1, the decision maker hires one of the experts based on their messages about their bias. In stage 2, this hired expert gets to see the true state and sends cheap talk message about the state. We ask two questions of this environment. One, does there exist a bias revealing equilibrium? The two sender environment is notably different from the one sender world in two ways - the introduction of sender competition (only one expert is hired while the other gets her reservation payoff) and strategic uncertainty (arising because the hiring decision depends upon the vector of bias announcements). Can these forces generate a bias revealing equilibrium when the one sender case could not? The second question of interest is whether the bias revealing equilibrium can give the decision maker a higher payoff than any equilibrium that arises in an environment without the bias messaging stage.

First, we show that in contrast to the one-sender case, truthful bias revelation in pure strategies is possible in an informative equilibrium in the two-sender world. We provide conditions under which such an equilibrium exists. Later, we consider an extension of the model where we illustrate through an example that there exists a bias revelation equilibrium which is strictly preferred by the decision maker to any equilibrium that can be achieved without the bias revealing stage. We begin with a numerical example and then show a general revelation result.

3.2.1 Example 2: Bias revealing equilibria exist

We use the same biases as in Example 1: suppose, $b_l = \frac{1}{6}$, $b_h = \frac{1}{5}$. Assume that $A_b = A = \frac{1}{12} + b_h^2 \, \forall b$ [7] and $P(b = b_h) = p_h \in (0,1)$. We show the following bias revealing strategies are part of a perfect Bayesian equilibrium.

Stage 1

$\mu_{ib}(x) = x \; \forall \; x \in \{b_l, b_h\} \; and \; i \in \{1,2\}$

$h(b_l, b_l) = \left(\frac{1}{2}, \frac{1}{2}\right)$

$h(b_l, b_h) = (1, 0)$

$h(b_h, b_l) = (0, 1)$

$h(b_h, b_h) = \left(\frac{1}{2}, \frac{1}{2}\right)$

Stage 2

If senders reports (b_l, b_l) in stage 1: Play a 2 partition CS b_l equilibrium with chosen expert

If senders reports (b_l, b_h) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_l) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_h) in stage 1: Play a 2 partition CS b_h equilibrium with chosen expert

Deviation by chosen expert in stage 2: take the lowest equilibrium action in 2 partition CS b_l equilibrium

Deviation by decision maker in hiring in stage 1: Play a babbling equilibrium with chosen expert

(9)

Clearly, neither the sender nor the decision maker has any incentives to deviate in stage 2 since the recommended strategies constitute an equilibrium in the stage 2 cheap talk game. Consider the decision maker's incentive to deviate in stage 1. The only payoff-relevant deviation is to hire a low bias expert when one expert announces a low bias and the other announces a high bias. However, since the decision maker's deviation will be detected and punished with a babbling equilibrium in stage 2, the decision maker cannot benefit from this deviation.

[7] So the individual rationality constraints are met for both types: every sender prefers to get hired than not.

Next, consider the incentives of a high bias expert. If she reports her bias truthfully, there are two possibilities: (a) if the other expert is also of high type, she is hired with probability $\frac{1}{2}$, and if hired, in stage 2, gets the payoff from a 2 partition CS b_h equilibrium. If she is not hired, she gets the reservation payoff of $-A$ (b) if the other expert if of low type, she is not hired and gets her reservation payoff of $-A$. Thus, using the formula from expression 1, we get that:

$$\textit{Payoff from truthfully reporting } b_h = p_h\left(\frac{1}{2}(-\frac{1}{12(2^2)} - \frac{b_h^2(2^2-1)}{3} - b_h^2) + \frac{1}{2}(-A)\right) - (1-p_h)A \tag{10}$$

If, on the other hand, the high bias expert lies about her bias and sends the bias message b_l, then there are two possibilities: (a) If the other expert is high bias, then she gets selected and gets the payoff from a babbling CS b_h equilibrium. (b) If the other expert is low type, then with probability half she does not get selected and gets the payoff of $-A$, and with probability half she gets selected and will hide in a two partition CS b_l equilibrium. Thus, using the formula from expression 2, we get that:

$$\textit{Payoff from lying and reporting } b_l = p_h(-A) + (1-p_h)(\frac{1}{2}(-A) + \frac{1}{2}(\frac{-1}{12(2)^2} + {b_l}^2(\frac{4}{3} - \frac{1}{2} - \frac{2^2}{3}) + b_l b_h \frac{2(1-2)}{2} - \frac{{b_h}^2}{2})) \tag{11}$$

Similarly, we can write the incentives of the low bias expert.

$$\textit{Payoff from truthfully reporting } b_l = p_h\left(\frac{1}{2}(-\frac{1}{12(2^2)} - \frac{b_l^2(2^2-1)}{3} - b_l^2) + \frac{1}{2}(-A)\right) - (1-p_h)A \tag{12}$$

$$\textit{Payoff from lying and reporting } b_h = (1-p_h)(-A) + p_h(\frac{1}{2}(-A) + \frac{1}{2}(\frac{-1}{12(2)^2} + {b_h}^2(\frac{4}{3} - \frac{1}{2} - \frac{2^2}{3}) + b_l b_h \frac{2(1-2)}{2} - \frac{{b_l}^2}{2})) \tag{13}$$

For the given parameters $b_l = \frac{1}{6}, b_h = \frac{1}{5}, A = \frac{1}{12} + b_h^2$, neither type of expert has an incentive to deviate from truth-telling if $p_h \in (0.61, 0.81)$. Thus, for this range of priors, truthful bias revelation can happen in a Perfect Bayesian equilibrium.

Next, we provide a sufficient condition under which bias revelation is possible in equilibrium.

3.2.2 General bias revealing equilibrium

Let n (respectively, m) be the highest partition cheap talk equilibrium possible when the sender's bias is known to be b_l (respectively, b_h). Since $b_h > b_l$, we know $n \geq m$. Let the payoff from not getting selected for the low bias sender be $-A_l = -A$, where $-A = -\frac{1}{12} - {b_l}^2$ is the babbling payoff for low type sender when there is no uncertainty about her type. If the high bias type sender does not get hired, she gets the payoff: $-A_h = -(A + c)$.

Consider the following strategy profile:

Stage 1

$\mu_{ib}(x) = x\ \forall\ x \in \{b_l, b_h\}\ and\ i \in \{1,2\}$

$h(b_l, b_l) = (\frac{1}{2}, \frac{1}{2})$

$h(b_l, b_h) = (0, 1)$

$h(b_h, b_l) = (1, 0)$

$h(b_h, b_h) = (\frac{1}{2}, \frac{1}{2})$

Stage 2

If senders reports (b_l, b_l) in stage 1: Play a n partition CS b_l equilibrium with chosen expert

If senders reports (b_l, b_h) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_l) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_h) in stage 1: Play a m partition CS b_h equilibrium with chosen expert

Deviation by chosen expert in stage 2: take the lowest equilibrium action in n partition CS b_l equilibrium

Deviation by decision maker in hiring in stage 1: Play a babbling equilibrium with chosen expert

(14)

The following proposition shows that this strategy profile constitutes a PBE, so bias revelation is supported in equilibrium.

Proposition 2. *There exists a p' such that if $p_h \in [0, p']$, then the above strategies are part of an informative Perfect Bayesian equilibrium.*

Proof. For a high bias sender:

$$\text{Payoff from announcing } b_h = p_h[\frac{1}{2}(-A-c) + \frac{CS^h_{b_h}(m)}{2}] + (1-p_h)(CS^h_{b_h}(1)) \tag{15}$$

$$\text{Payoff from announcing } b_l = p_h(-A-c) + (1-p_h)[\frac{1}{2}(-A-c) + \frac{CS^h_{b_l}(n)}{2}] \tag{16}$$

Thus, to satisfy this IC, we must have:

$$p_h \geq \frac{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) - \frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) + \frac{1}{2}CS^h_{b_h}(m)}$$

Note that for a feasible solution to exist, we need

$$\frac{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) - \frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) + \frac{1}{2}CS^h_{b_h}(m)} \leq 1$$

Since $\frac{1}{2}CS^h_{b_h}(m) > -\frac{A+c}{2}$[8], this is always satisfied.

Next, consider the incentives for a low bias sender:

$$\text{Payoff from announcing } b_h = p_h[\frac{1}{2}(-A) + \frac{CS^l_{b_h}(m)}{2}] + (1-p_h)CS^l_{b_l}(1) \tag{17}$$

$$\text{Payoff from announcing } b_l = p_h(-A) + (1-p_h)[\frac{1}{2}(-A) + \frac{CS^l_{b_l}(n)}{2}] \tag{18}$$

This incentive constraint is satisfied if:

$$p_h \leq \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}} \tag{19}$$

[8] As the payoff from being hired is always designed to be greater than not being hired.

Note that for a feasible solution to exist, we need

$$\frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}} \geq 0$$

Now, the numerator

$$\frac{CS^l_{b_l}(n) - CS^l_{b_l}(1)}{2} \geq 0$$

since $-A = CS^l_{b_l}(1)$, and $CS^l_{b_l}(n) \geq CS^l_{b_l}(1)$ since $n \geq 1$. The denominator can be rewritten as:

$$\begin{aligned} & \frac{CS^l_{b_l}(n) - CS^l_{b_l}(1)}{2} + \frac{CS^l_{b_h}(m) - CS^l_{b_l}(1)}{2} \\ \geq \quad & CS^l_{b_h}(m) - CS^l_{b_l}(1) \quad \left(\sin ce\ n \geq m \Rightarrow CS^l_{b_l}(n) \geq CS^l_{b_h}(m)\right) \end{aligned}$$

This holds if ${b_h}^2 < \frac{1}{4m^2}$. We know this is true because $b_h < \frac{1}{2m(m-1)}$ (to guarantee an m partition equilibrium in the CS world for the sender's bias equal to b_h).

So, there is a range of p_h for which both types' incentive constraints hold only if:

$$\frac{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) - \frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) + \frac{CS^h_{b_h}(m)}{2}} \leq \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}}$$

For $c > (CS^h_{b_l}(n) - CS^h_{b_h}(1)) + (CS^l_{b_l}(1) - CS^h_{b_h}(1))$, the LHS is negative. We have already shown that the right-hand side is positive. Thus, the ICs can be satisfied simulteneously for a range of p_h.

Equating the IC for the low bias sender, we find

$$p' = \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}}$$

such that if $p_h \in [0, p']$ then bias revelation is an equilibrium for the strategies given in 14. □

3.2.3 Welfare

In this subsection, we want to determine conditions under which a revealing equilibrium is payoff superior to any equilibrium possible without the bias revealing stage. First, we notice that while the equilibrium illustrated in proposition 2 allows for endogenous bias revelation, it fares poorly on receiver welfare owing to the babbling equilibrium (which gives the worst equilibrium payoff) being played when the senders' messages don't match in stage 1. There is a trade-off between bias revelation and decision maker's payoff maximization: in stage 2, if the most informative equilibrium is selected, that will enhance the utility of the decision maker, but in stage 1, the incentive constraints will not be satisfied, especially for the high bias type sender, if that most informative equilibrium is played later. The issue is that if we pick either type with probability one after mixed messages (when the senders' reports about bias do not match), we are unable to generate incentives for truth-telling for both types. On the other hand, if we let the receiver use a non-deterministic hiring strategy after mixed messages and this is followed by the most informative equilibria then the receiver will choose to deviate from the non-deterministic hiring strategy and will always pick the low-type sender after mixed messages in any bias revealing equilibrium.

To mitigate this we allow the receiver to use a public randomization device to commit to mixed strategies as a response to bias announcements. Any deviations by the receiver will now be observable and can be punished with a babbling equilibrium.

First, we consider the one sender case again and show that allowing the receiver the ability to commit to mixing on hiring does not alter the result there. The following result shows there cannot be a bias revealing equilibrium in the one sender case even after allowing for mixed strategy in hiring by the receiver.

Proposition 3. *There does not exist any informative bias revealing equilibrium with one sender.*

Proof. Consider the following strategies. Both types of senders reveal their type. For the decision maker's strategy, we abuse notation and write the equilibrium she will play:

Decision maker strategy:

If DM observes b_l in stage $1 = (v_l)$Play j partition CS b_l eq $+ (1-v_l)$Play k partition CS b_l eq

If DM observes b_h in stage $1 = (v_h)$Play x partition CS b_h eq $+ (1-v_h)$Play y partition CS b_h eq

where $j,k,x,y \in \mathbb{N}$ and $v_l, v_h \in [0,1]$. WLOG let $j \geq k, x \geq y$

Suppose such an equilibrium exists for some choice of parameters. The IC conditions for truth-telling is give by,

$$IC_{b_l}:\ v_l CS^l_{b_l}(j) + (1-v_l)CS^l_{b_l}(k) \geq v_h CS^l_{b_h}(x) + (1-v_h)CS^l_{b_h}(y) \tag{20}$$

$$IC_{b_h}:\ v_h CS^h_{b_h}(x) + (1-v_h)CS^h_{b_h}(y) \geq v_l CS^h_{b_l}(j) + (1-v_l)CS^h_{b_l}(k) \tag{21}$$

Adding, we get,

$$\begin{aligned} &v_l(CS^l_{b_l}(j) - CS^h_{b_l}(j)) + (1-v_l)(CS^l_{b_l}(k) - CS^h_{b_l}(k)) \geq \\ &v_h(CS^l_{b_h}(x) - CS^h_{b_h}(x)) + (1-v_h)(CS^l_{b_h}(y) - CS^h_{b_h}(y)) \end{aligned} \tag{22}$$

Using equations 1, 2 and the corresponding expressions for the low bias sender, we can calculate:

$$CS^l_{b_l}(n) - CS^h_{b_l}(n) = (b_h - b_l)(b_h \frac{1}{n} + b_l(2 - \frac{1}{n})) \tag{23}$$

$$CS^l_{b_h}(m) - CS^h_{b_h}(m) = (b_h - b_l)(b_h(2 - \frac{1}{m}) + b_l \frac{1}{m}) \tag{24}$$

Plugging in the values from above in 22 we get,

$$(b_h - b_l)\left(\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} - 2\right) \geq 0 \tag{25}$$

Since $b_h \geq b_l$ this would be true if and only if

$$\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} \geq 2 \tag{26}$$

which would require $j, k, x, y \leq 1$. This would imply the equilibrium play would be babbling under all possible revelation in the first stage. However, this is not true since we were looking for an informative bias revealing equilibrium. Therefore our assumption is wrong and there does not exist any informative equilibrium in the one sender case even if we permit the receiver to mix with commitment in hiring. $\square$

Next, we turn to the two sender case and show an example where under some conditions, not only does a bias-revealing equilibrium exist, but it gives the decision maker a higher utility than any equilibrium that can be achieved without the bias-revealing stage 1.

First we will find the payoff maximizing bias revealing equilibrium. Then, we will construct an example in which this equilibrium does better than the best possible equilibrium when there is no bias revealing stage.

Consider the following bias revealing strategy profile:

Stage 1

$\mu_{ib}(x) = x \; \forall \; x \in \{b_l, b_h\} \; and \; i \in \{1,2\}$

$h(b_l, b_l) = (\frac{1}{2}, \frac{1}{2})$

$h(b_l, b_h) = (1-v, v) \;, v \in [0,1]$

$h(b_h, b_l) = (v, 1-v)$

$h(b_h, b_h) = (\frac{1}{2}, \frac{1}{2})$

Stage 2

If senders reports (b_l, b_l) in stage 1: Play a n partition CS b_l equilibrium with chosen expert

If senders reports (b_l, b_h) or (b_h, b_l) in stage 1: If b_l chosen - play a n partition CS b_l equilibrium, If b_h chosen - play a m partition CS b_h equilibrium

If senders reports (b_h, b_h) in stage 1: Play a m partition CS b_h equilibrium with chosen expert

Deviation by chosen expert in stage 2: take the lowest equilibrium action in n partition CS b_l equilibrium

Deviation by decision maker in hiring in stage 1: Play a babbling equilibrium with chosen expert

(27)

We will construct an example where this strategy profile can be supported as a PBE.

IC for b_h:

$$p_h > \frac{\frac{1}{2}(CS^h_{b_l}(n) - CS^h_{b_h}(m)) + (\frac{1}{2} - v)(CS^h_{b_h}(m) + A + c)}{(v - \frac{1}{2})(CS^h_{b_l}(n) - CS^h_{b_h}(m))}$$

We need that the RHS is less than 1 to get a feasible region for p_h.

Consider $v > \frac{1}{2}$. Then, denominator is positive, since $CS^h_{b_l}(n) \geq CS^h_{b_h}(m)$. In the numerator, since $CS^h_{b_l}(n) \geq CS^h_{b_h}(m)$, and $CS^h_{b_h}(m) > -(A+c)$, we have the first expression to be positive while the second expression to be negative. We can increase c to make sure that the RHS is less than 1. For every v, let $c_1(v)$ be the lowest positive real number such that for that v, for all $c > c_1(v) \Rightarrow RHS < 1$.

IC for b_l:

$$p_h < \frac{\frac{1}{2}(CS^l_{b_l}(n) - CS^l_{b_h}(m)) + (\frac{1}{2} - v)(CS^l_{b_h}(m) + A)}{(v - \frac{1}{2})(CS^l_{b_l}(n) - CS^l_{b_h}(m))}$$

We need that the RHS is greater than zero to get a feasible region for p_h. Again consider $v > \frac{1}{2}$. Then, denominator is positive. Further, the first expression of numerator is positive while the second expression is negative. Note that when v is close to $\frac{1}{2}$ from above, RHS is positive. Let v_1 be the highest v above $\frac{1}{2}$ such that $\frac{1}{2} < v < v_1 \Rightarrow RHS > 0$.

To satisfy both the ICs simultaneously, we need:

$$\frac{\frac{1}{2}(CS^h_{b_l}(n) - CS^h_{b_h}(m)) + (\frac{1}{2} - v)(CS^h_{b_h}(m) + A + c)}{(v - \frac{1}{2})(CS^h_{b_l}(n) - CS^h_{b_h}(m))} < \frac{\frac{1}{2}(CS^l_{b_l}(n) - CS^l_{b_h}(m)) + (\frac{1}{2} - v)(CS^l_{b_h}(m) + A)}{(v - \frac{1}{2})(CS^l_{b_l}(n) - CS^l_{b_h}(m))}$$

Choose a v'' such that $\frac{1}{2} < v'' < v_1$. For each v in this range, we can always choose c high enough to make the above inequality hold. Suppose it holds when $c > c_2\left(v''\right)$. Now for this v'' $(> \frac{1}{2})$, we can find $c_1\left(v''\right)$ which makes the IC for b_h feasible.

Then, for $\frac{1}{2} < v'' < v_1$, we can find a $c > max\left\{c_2\left(v''\right), c_1\left(v''\right)\right\}$ such that our bias revealing strategy profile is an equilibrium.

Consider an example where $b_h = 0.2$ and $b_l = 0.072$ (notice that $1/24 < b_l < 1/12$). Thus, when biases are known a three partition equilibrium is possible with the low bias sender but the high bias allows a maximum of 2 partitions.

Payoff for decision receiver in this best revealing equilibrium =

$$\begin{aligned} &p_h^2(-\frac{1}{48} - b_h^2) + (1-p_h)^2\left(-\frac{1}{12\cdot 3^2} - \frac{b_l^2\cdot(3^2-1)}{3}\right) + \\ &2p_h(1-p_h)(v\left(-\frac{1}{48} - b_h^2\right) + (1-v)\left(-\frac{1}{12\cdot 3^2} - \frac{b_l^2\cdot(3^2-1)}{3}\right)) \end{aligned} \tag{28}$$

In particular, if $v = 0.65$ and $c = 0.17$, for $p_h \in (0.068, 0.092)$, this bias revealing strategy profile constitutes a PBE.

Now, let us compare this payoff to the maximum payoff that can be achieved without a bias revealing stage. First, we find conditions under which the receiver payoff maximizing equilibrium when there is no bias revealing stage 1 can sustain a maximum of two partitions. Intuitively, if p_h is high enough this will hold. Then, we find conditions under which this equilibrium does worse than the equilibrium specified by the bias revealing strategies in 27.

Note that without a bias revealing stage, the decision maker has no way of differentiating between the two senders and will therefore pick one at random. Subsequently, the chosen expert and decision maker will play a cheap talk game where the sender's bias is uncertain, a la the non-disclosure world of Li and Madarász (2008).

For a three partition equilibrium without the bias revealing stage, the cutoffs for the high type are:

$$a_1 = \frac{1}{3} + 3d(1-p_h) - 4b_h$$
$$a_2 = \frac{2}{3} + 3d(1-p_h) - 4b_h$$

and the low bias sender's cutoffs are a_1+d, a_2+d where $d = b_h - b_l$. The high type sends message m_1 in $[0,a_1)$, m_2 in $[a_1,a_2)$ and m_3 in $[a_2,1]$, and the low bias sender sends messages m_1 in $[0,a_1+d)$, m_2 in $[a_1+d,a_2+d)$ and m_3 in $[a_2+d,1]$. Note that if $a_1 < 0$ and $a_1+d > 0$, then we could have a three partition equilibrium where the bias of the low type is endogenously revealed in equilibrium for the states which lie between zero and a_1+d.[9] Suppose $a_1+d \leq 0$. Then, the maximum number of partitions possible in the world without a bias revealing stage is two. This is because if $a_1+d \leq 0$ then $a_1 < 0$, thus in the *LM* equilibrium both types will choose exactly two partitions if

$$a_1 + d \leq 0 \;\Rightarrow\; p_h > \frac{(\frac{1}{3} - 4b_l)}{3d}$$

Payoff for the decision maker in any two partition equilibrium without the bias revealing stage:

$$-p_h \int_0^{a_h} (y_1-t)^2 dt - (1-p_h) \int_0^{a_l} (y_1-t)^2 dt - p_h \int_{a_h}^1 (y_2-t)^2 dt - (1-p) \int_{a_l}^1 (y_2-t)^2 dt \qquad (29)$$

where $a_h = \frac{1}{2} + (1-p_h)d - 2b_h$, $a_l = a_h + d$, $y_1 = \frac{1}{4} - p_h d - b_l$, $y_2 = \frac{3}{4} - pd - b_l$

Now, we can check that if $v = 0.65$, and $p_h \in (0.068, 0.092)$, $A_{b_l} = \frac{1}{12} + b_l^2$, $A_{b_h} = A_{b_l} + 0.17$, then the conditions required for a bias revealing equilibrium are met and the bias revealing equilibrium we highlight gives the decision maker a higher payoff than that possible in any equilibrium in the LM world.

[9] Li and Madarász (2008) speak about the possibility of such conflict revealing equilibrium.

4 Conclusion

In this paper, we consider a model of strategic information transmission where the experts have uncertain biases and may choose to disclose them before communicating state-relevant information. We build on the framework developed in Li and Madarász (2008) and make bias revelation an endogenous choice. We find that if there is only one sender, full revelation of bias is not possible in equilibrium. With two senders, we identify conditions for a bias-revealing equilibrium to exist. Moreover, we find that if the players have access to a public randomization device, then the decision maker could be better off with a bias-revealing equilibrium compared to the best equilibrium possible with no bias revelation.

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